

# WORKSHOP CALCULATION & SCIENCE

(NSQF)

(As per Revised Syllabus July 2022)

**CARPENTER**



Directorate General of Training

DIRECTORATE GENERAL OF TRAINING  
MINISTRY OF SKILL DEVELOPMENT & ENTREPRENEURSHIP  
GOVERNMENT OF INDIA



**NATIONAL INSTRUCTIONAL  
MEDIA INSTITUTE, CHENNAI**

Post Box No. 3142, CTI Campus, Guindy, Chennai - 600 032

**Workshop Calculation & Science**  
**Carpenter - 1 Year NSQF**  
**As per Revised Syllabus July 2022**

**Developed & Published by**



**National Instructional Media Institute**

Post Box No.3142  
Guindy, Chennai - 600032  
INDIA  
Email: [chennai-nimi@nic.in](mailto:chennai-nimi@nic.in)  
Website: [www.nimi.gov.in](http://www.nimi.gov.in)

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## FOREWORD

The Government of India has set an ambitious target of imparting skills to 30 crores people, one out of every four Indians, by 2020 to help them secure jobs as part of the National Skills Development Policy. Industrial Training Institutes (ITIs) play a vital role in this process especially in terms of providing skilled manpower. Keeping this in mind, and for providing the current industry relevant skill training to Trainees, ITI syllabus has been recently updated with the help of comprising various stakeholder's viz. Industries, Entrepreneurs, Academicians and representatives from ITIs.

The National Instructional Media Institute (NIMI), Chennai, has now come up with instructional material to suit the revised curriculum for **Workshop Calculation & Science - Carpenter NSQF (Revised 2022)** under CTS will help the trainees to get an international equivalency standard where their skill proficiency and competency will be duly recognized across the globe and this will also increase the scope of recognition of prior learning. NSQF trainees will also get the opportunities to promote life long learning and skill development. I have no doubt that with NSQF the trainers and trainees of ITIs, and all stakeholders will derive maximum benefits from these IMPs and that NIMI's effort will go a long way in improving the quality of Vocational training in the country.

The Executive Director & Staff of NIMI and members of Media Development Committee deserve appreciation for their contribution in bringing out this publication.

Jai Hind

New Delhi - 110 001

**Ms. TRISHALJIT SETHI,**  
Additional Secretary / Director General (Training),  
Ministry of Skill Development & Entrepreneurship,  
Government of India.

## PREFACE

The National Instructional Media Institute(NIMI) was set up at Chennai, by the Directorate General of Training, Ministry of skill Development and Entrepreneurship, Government of India, with the technical assistance from the Govt of the Federal Republic of Germany with the prime objective of developing and disseminating instructional Material for various trades as per prescribed syllabus and Craftsman Training Programme(CTS) under NSQF levels.

The Instructional materials are developed and produced in the form of Instructional Media Packages (IMPs), consisting of Trade Theory, Trade Practical, Test and Assignment Book, Instructor Guide, Wall charts, Transparencies and other supportive materials. The above material will enable to achieve overall improvement in the standard of training in ITIs.

A national multi-skill programme called SKILL INDIA, was launched by the Government of India, through a Gazette Notification from the Ministry of Finance (Dept of Economic Affairs), Govt of India, dated 27th December 2013, with a view to create opportunities, space and scope for the development of talents of Indian Youth, and to develop those sectors under Skill Development.

The emphasis is to skill the Youth in such a manner to enable them to get employment and also improve Entrepreneurship by providing training, support and guidance for all occupation that were of traditional types. The training programme would be in the lines of International level, so that youths of our Country can get employed within the Country or Overseas employment. The **National Skill Qualification Framework (NSQF)**, anchored at the National Skill Development Agency(NSDA), is a Nationally Integrated Education and competency-based framework, to organize all qualifications according to a series of **levels of Knowledge, Skill and Aptitude**. Under NSQF the learner can acquire the Certification for Competency needed at any level through formal, non-formal or informal learning.

The **Workshop Calculation & Science** - Carpenter NSQF (Revised 2022) under CTS is one of the book developed by the core group members as per the NSQF syllabus.

The **Workshop Calculation & Science** - Carpenter NSQF (Revised 2022) under CTS as per NSQF is the outcome of the collective efforts of experts from Field Institutes of DGT, Champion ITI's for each of the Sectors, and also Media Development Committee (**MDC**) members and Staff of **NIMI**. NIMI wishes that the above material will fulfill to satisfy the long needs of the trainees and instructors and shall help the trainees for their Employability in Vocational Training.

NIMI would like to take this opportunity to convey sincere thanks to all the Members and Media Development Committee (MDC) members.

Chennai - 600 032

EXECUTIVE DIRECTOR



## ACKNOWLEDGEMENT

The National Instructional Media Institute (NIMI) sincerely acknowledge with thanks the co-operation and contribution of the following Media Developers to bring this IMP for the course **Workshop Calculation & Science - Carpenter** as per NSQF Revised 2022.

### MEDIA DEVELOPMENT COMMITTEE MEMBERS

|                          |   |   |
|--------------------------|---|---|
| Shri. M. Sangara pandian | - | Training Officer (Retd.)<br>CTI, Govt. of India, Guindy, Chennai - 32.    |
| Shri. G. Sathiamoorthy   | - | Jr. Training Officer - SG (Retd.)<br>Govt I.T.I, Trichy, DET - Tamilnadu. |

### NIMI CO-ORDINATORS

|                        |   |  |
|------------------------|---|--|
| Shri. Nirmalya Nath    | - | Deputy General Manager,<br>NIMI, Chennai - 32. |
| Shri. G. Michael Johny | - | Manager,<br>NIMI, Chennai - 32.                |

NIMI records its appreciation of the **Data Entry, CAD, DTP Operators** for their excellent and devoted services in the process of development of this IMP.

NIMI also acknowledges with thanks, the efforts rendered by all other staff who have contributed for the development of this book.

## INTRODUCTION

The material has been divided into independent learning units, each consisting of a summary of the topic and an assignment part. The summary explains in a clear and easily understandable fashion the essence of the mathematical and scientific principles. This must not be treated as a replacement for the instructor's explanatory information to be imparted to the trainees in the classroom, which certainly will be more elaborate. The book should enable the trainees in grasping the essentials from the elaboration made by the instructor and will help them to solve independently the assignments of the respective chapters. It will also help them to solve the various problems, they may come across on the shop floor while doing their practical exercises.

The assignments are presented through 'Graphics' to ensure communications amongst the trainees. It also assists the trainees to determine the right approach to solve the problems. The required relevant data to solve the problems are provided adjacent to the graphics either by means of symbols or by means of words. The description of the symbols indicated in the problems has its reference in the relevant summaries.

At the end of the exercise wherever necessary assignments, problems are included for further practice.

**Time allotment:**

**Duration of 1 Year : 26 Hrs**

Time allotment for each title of exercises has been given below. **Workshop Calculation & Science - Carpenter** NSQF Revised Syllabus 2022.

| S.No         | Title  | Exercise No.    | Time in Hrs   |
|--------------|--|-----------------|---------------|
| 1            | Unit, Fractions                                | 1.1.01 - 1.1.07 | 4             |
| 2            | Square root, Ratio and Proportions, Percentage | 1.2.08 - 1.2.14 | 6             |
| 3            | Material Science                               | 1.3.15 - 1.3.17 | 4             |
| 4            | Mass, Weight, Volume and Density               | 1.4.18 & 1.4.19 | 4             |
| 5            | Mensuration                                    | 1.5.20 - 1.5.23 | 6             |
| 6            | Trigonometry                                   | 1.6.24 & 1.6.25 | 2             |
| <b>Total</b> |  |                 | <b>26 Hrs</b> |

### LEARNING / ASSESSABLE OUTCOME

On completion of this book you shall be able to

- **Demonstrate basic mathematical concept and principles to perform practical operations.**
- **Understand and explain basic science in the field of study.**

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# SYLLABUS

1 Year

## Workshop Calculation & Science - Carpenter Revised syllabus July 2022 under CTS

| S.No.      | Title   | Time in Hrs |
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|            | <b>Total</b>  | <b>26</b>   |

Unit, Fractions - Classification of unit system

**Necessity**

All physical quantities are to be measured in terms of standard quantities.

**Unit**

A unit is defined as a standard or fixed quantity of one kind used to measure other quantities of the same kind.

**Classification**

Fundamental units and derived units are the two classifications.

**Fundamental units**

Units of basic quantities of length, mass and time.

**Derived units**

Units which are derived from basic units and bear a constant relationship with the fundamental units. E.g. area, volume, pressure, force etc.

**Systems of units**

- F.P.S system is the British system in which the basic units of length, mass and time are foot, pound and second respectively.
- C.G.S system is the metric system in which the basic units of length, mass and time are centimeter, gram and seconds respectively.
- M.K.S system is another metric system in which the basic units of length, mass and time are metre, kilogram and second respectively.
- S.I. units are referred to as Systems International units which is again of metric and the basic units, their names and symbols are as follows.

Fundamental units and derived units are the two classifications of units.

Length, mass and time are the fundamental units in all the systems (i.e) F.P.S, C.G.S, M.K.S and S.I. systems.

Example

**Length:** What is the length of copper wire in the roll, if the roll of copper wire weighs 8kg, the dia of wire is 0.9cm and the density is 8.9 gm/cm<sup>3</sup>?

**Solution**

mass of copper wire in the roll = 8kg (or) 8000grams

Dia of copper wire in the roll = 0.9cm

Density of copper wire = 8.9 gm/cm<sup>3</sup>

Area of cross section of copper wire

$$= \frac{\pi d^2}{4} = \frac{\pi \times (0.9^2)}{4} = 0.636cm^2$$

Volume of copper wire

$$= \frac{\text{Mass of copper wire}}{\text{Density of copper wire}} = \frac{8000\text{grams}}{8.9 \text{ gm/cm}^3} = 898.88cm^3$$

Length of copper wire

$$= \frac{\text{Volume of copper wire}}{\text{Area of cross section of copper wire}} = \frac{898.88cm^3}{0.636cm^2} = 1413.33 \text{ cm}$$

Length of copper wire = 1413cm.

**Time:** The S.I. unit of time, the second, is another base units of S.I., it is defined as the time interval occupied by a number of cycles of radiation from the calcium atom. The second is the same quantity in the S.I. in the British and in the U.S. systems of units.

**Fundamental units of F.P.S, C.G.S, M.K.S and S.I**

| S.No. | Basic quantity  | British units |        | Metric units |        |            |        | International units |        |
|-------|-----------------|---------------|--------|--------------|--------|------------|--------|---------------------|--------|
|       |                 | F.P.S         | Symbol | C.G.S        | Symbol | M.K.S      | Symbol | S.I Units           | Symbol |
| 1     | Length          | Foot          | ft     | Centimetre   | cm     | Metre      | m      | Metre               | m      |
| 2     | Mass            | Pound         | lb     | Gram         | g      | Kilogram   | kg     | Kilogram            | Kg     |
| 3     | Time            | Second        | s      | Second       | s      | Second     | s      | Second              | s      |
| 4     | Current         | Ampere        | A      | Ampere       | A      | Ampere     | A      | Ampere              | A      |
| 5     | Temperature     | Fahrenheit    | °F     | Centigrade   | °C     | Centigrade | °C     | Kelvin              | K      |
| 6     | Light intensity | Candela       | Cd     | Candela      | Cd     | Candela    | Cd     | Candela             | Cd     |

## Unit, Fractions - Fundamental and Derived units F.P.S, C.G.S, M.K.S and SI units

Derived units of F.P.S, C.G.S, M.K.S and SI system

| S.No | Physical quantity | British units          |                    | Metric units                 |                     |                           |                    | International units      |                          |
|------|-------------------|------------------------|--------------------|------------------------------|---------------------|---------------------------|--------------------|--------------------------|--------------------------|
|      |                   | FPS                    | Symbol             | CGS                          | Symbol              | MKS                       | Symbol             | SIUnits                  | Symbol                   |
| 1    | Area              | Square foot            | ft <sup>2</sup>    | Square centimetre            | cm <sup>2</sup>     | Square metre              | m <sup>2</sup>     | Square metre             | m <sup>2</sup>           |
| 2    | Volume            | Cubic foot             | ft <sup>3</sup>    | Cubic centimetre             | cm <sup>3</sup>     | Cubic metre               | m <sup>3</sup>     | Cubic metre              | m <sup>3</sup>           |
| 3    | Density           | Pound per cubic foot   | lb/ft <sup>3</sup> | Gram per cubic centimetre    | g/cm <sup>3</sup>   | Kilogram per cubic metre  | kg/m <sup>3</sup>  | Kilogram per cubic metre | Kg/m <sup>3</sup>        |
| 4    | Speed             | Foot per second        | ft/s               | Centimetre per second        | cm/sec              | Metre per second          | m/sec              | Metre per second         | m/sec                    |
| 5    | Velocity (linear) | Foot per second        | ft/s               | Centimetre per second        | cm/sec              | Metre per second          | m/sec              | Metre per second         | m/sec                    |
| 6    | Acceleration      | Foot per square second | ft/s <sup>2</sup>  | Centimetre per square second | cm/sec <sup>2</sup> | Metre per square second   | m/sec <sup>2</sup> | Metre per square second  | m/sec <sup>2</sup>       |
| 7    | Retardation       | Foot per square Second | ft/s <sup>2</sup>  | Centimetre per square second | cm/sec <sup>2</sup> | Metre per square second   | m/sec <sup>2</sup> | Metre square second      | m/sec <sup>2</sup>       |
| 8    | Angular velocity  | Degree per second      | Deg/sec            | Radian per second            | rad/sec             | Radian per second         | rad/sec            | Radian per second        | rad/sec                  |
| 9    | Mass              | Pound (slug)           | lb                 | Gram                         | g                   | Kilogram                  | kg                 | Kilogram                 | kg                       |
| 10   | Weight            | Pound                  | lb                 | Gram                         | g                   | Kilogram weight           | kg                 | Newton                   | N                        |
| 11   | Force             | Pounds                 | lbf                | dyne                         | dyn                 | Kilogram force            | kgf                | Newton                   | N(kgm/sec <sup>2</sup> ) |
| 12   | Power             | Foot pound per second  | ft.lb/sec          | Gram.centimetre/sec          | g.cm/sec            | kilogram metre per second | kg.m/sec           | -                        | -                        |
|      |                   | Horse power            | hp                 | Erg per second               |                     | watt                      | W                  | watt                     | W(J/sec)                 |
| 13   | Pressure, Stress  | Pound per square inch  | lb/in <sup>2</sup> | Gram per square centimetre   | g/cm <sup>2</sup>   | Kilogram per square metre | kg/m <sup>2</sup>  | Newton per square metre  | N/m <sup>2</sup>         |
| 14   | Energy, Work      | Foot.pound             | ft.lb              | Gram centimetre              | g.cm                | Kilogram metre            | kg.m               | joule                    | J(Nm)                    |
| 15   | Heat              | British thermal unit   | BTU                | calorie                      | Cal                 | joule                     | J                  | joule                    | J(Nm)                    |
| 16   | Torque            | Pound force foot       | lbf.ft             | Newton millimetre            | N mm                | Kilogram metre            | kg.m               | Newton metre             | Nm                       |
| 17   | Temperature       | Degree Fahrenheit      | °F                 | Degree Centigrade            | °C                  | Kelvin                    | K                  | Kelvin                   | K                        |

Unit, Fractions - Measurement units and conversion

Units and abbreviations

| Quantity                  | Units   | Abbreviation of unit                        |
|---------------------------|---|---|
| Calorific value           | kilojoules per kilogram   | kJ/kg                                       |
| Specific fuel consumption | kilogram per hour per newton  | kg/hr/N                                     |
| Length                    | millimetre, metre, kilometre  | mm, m, km                                   |
| Mass                      | kilogram, gram  | kg, g                                       |
| Time                      | seconds, minutes, hours   | s, min, h                                   |
| Speed                     | centimetre per second,<br>metre per second<br>kilometre per hour, miles per hour                    | cm/s, m/s<br>km/h, mph                      |
| Acceleration              | metre-per-square second   | m/s <sup>2</sup>                            |
| Force                     | newtons, kilonewtons  | N, kN                                       |
| Moment                    | newton-metres   | Nm  |
| Work                      | joules  | J   |
| Power                     | horsepower, watts, kilowatts  | Hp, W, kW                                   |
| Pressure                  | newton per square metre<br>kilonewton per square metre  | N/m <sup>2</sup><br>kN/m <sup>2</sup>       |
| Angle                     | radian  | rad   |
| Angular speed             | radians per second<br>radians-per-square second<br>revolutions per minute<br>revolutions per second | rad/s<br>rad/s <sup>2</sup><br>Rpm<br>rev/s |

Decimal multiples and parts of unit

| Decimal power     | Value          | Prefixes | Symbol | Stands for           |
|-------------------|----------------|----------|--------|----------------------|
| 10 <sup>12</sup>  | 1000000000000  | tera     | T      | billion times        |
| 10 <sup>9</sup>   | 1000000000     | giga     | G      | thousand millintimes |
| 10 <sup>6</sup>   | 1000000        | mega     | M      | million times        |
| 10 <sup>3</sup>   | 1000           | kilo     | K      | thousand times       |
| 10 <sup>2</sup>   | 100            | hecto    | h      | hundred times        |
| 10 <sup>1</sup>   | 10             | deca     | da     | ten times            |
| 10 <sup>-1</sup>  | 0.1            | deci     | d      | tenth                |
| 10 <sup>-2</sup>  | 0.01           | centi    | c      | hundredth            |
| 10 <sup>-3</sup>  | 0.001          | milli    | m      | thousandth           |
| 10 <sup>-6</sup>  | 0.000001       | micro    | μ      | millionth            |
| 10 <sup>-9</sup>  | 0.000000001    | nano     | n      | thousand millionth   |
| 10 <sup>-12</sup> | 0.000000000001 | pico     | p      | billionth            |

**SI units and the British units:**

| Quantity                  | SI unit → British unit  | British unit → SI unit   |
|---------------------------|---|--|
| Length                    | 1 m = 3.281 ft<br>1 km = 0.621 mile   | 1 ft = 0.3048 m<br>1 mile = 1.609 km   |
| Speed                     | 1 m/s = 3.281 ft/s<br>1 km/h = 0.621 mph  | 1 ft/s = 0.305 m/s<br>1 mph = 1.61 km/h  |
| Acceleration              | 1 m/s <sup>2</sup> = 3.281 ft/s <sup>2</sup>  | 1 ft/s <sup>2</sup> = 0.305 m/s <sup>2</sup>   |
| Mass                      | 1 kg = 2.205 lb   | 1 lb = 0.454 kg  |
| Force                     | 1 N = 0.225 lbf<br>1 MN   | 1 lbf = 4.448 N<br>1 million newtons   |
| Torque                    | 1 Nm = 0.738 lbf ft   | 1 lbf ft = 1.355 Nm  |
| Pressure                  | 1 N/m <sup>2</sup> = 0.000145 lbf/in <sup>2</sup><br>1 Pa = 1 N/m <sup>2</sup><br>1 bar = 14.5038 lbf/in <sup>2</sup> | 1 lbf/in <sup>2</sup> = 6.896 kN/m <sup>2</sup><br>1 lbf/in <sup>2</sup> = 6.895 kN/m <sup>2</sup> |
| Energy, work              | 1 J = 0.738 ft lbf<br>1 J = 0.239 calorie<br>1 kJ = 0.948 BTU<br>(1 therm = 100 000 BTU)<br>1 kJ = 0.526 CHU          | 1 ft lbf = 1.355 J<br>1 calorie = 4.186 J<br>1 BTU = 1.055 kJ<br>1 CHU = 1.9 kJ                    |
| Power                     | 1 kW = 1.34 hp  | 1 hp = 0.7457 kW   |
| Fuel consumption          | 1 km/L = 2.82 mile/gallon   | 1 mpg = 0.354 km/L   |
| Specific fuel consumption | 1 kg/kWh = 1.65 lb/bhp h<br>1 litre/kWh = 1.575 pt/bhp h  | 1 lb/bhp h = 0.606 kg/kWh<br>1 pt/bhp h = 0.631 litre/kWh  |
| Calorific value           | 1 kJ/kg = 0.43 BTU/lb<br>1 kJ/kg = 0.239 CHU/lb   | 1 BTU/lb = 2.326 kJ/kg<br>1 CHU/lb = 4.188 kJ/kg   |



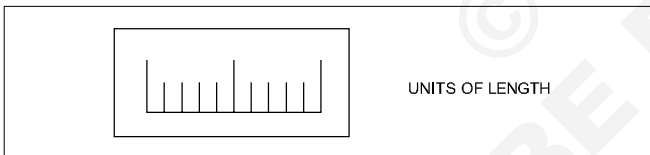
## Prefixes for decimal multiples and submultiples

| Use          |                          |
|--------------|--------------------------|
| 1 Megapascal | = 1 MPa = 1000000 Pa     |
| 1 Kilowatt   | = 1 kW = 1000 W          |
| 1 Hectolitre | = 1 hL = 100 L           |
| Decanewton   | = 1 daN = 10 N           |
| Decimetre    | = 1 dm = 0.1 m           |
| 1 Centimetre | = 1 cm = 0.01 m          |
| 1 Millimetre | = 1 mm = 0.001 m         |
| 1 Micrometre | = 1 $\mu$ m = 0.000001 m |

## Conversion factors

|              |                |
|--------------|----------------|
| 1 inch       | = 25.4 mm      |
| 1 mm         | = 0.03937 inch |
| 1 metre      | = 39.37 inch   |
| 1 micron     | = 0.00003937"  |
| 1 kilometre  | = 0.621 miles  |
| 1 pound      | = 453.6 g      |
| 1 kg         | = 2.205 lbs    |
| 1 metric ton | = 0.98 ton     |

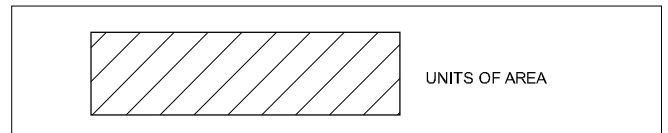
## Units of physical quantities



### Units of length

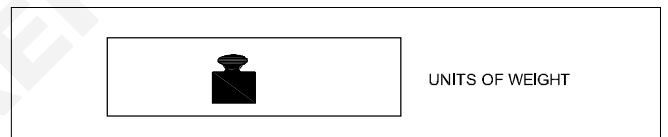
|                   |         |              |
|-------------------|---------|--------------|
| Micron            | 1 $\mu$ | = 0.001 mm   |
| Millimetre        | 1 mm    | = 1000 $\mu$ |
| Centimetre        | 1 cm    | = 10 mm      |
| Decimetre         | 1 dm    | = 10 cm      |
| Metre             | 1 m     | = 10 dm      |
| Kilometre         | 1 km    | = 1000 m     |
| Inch              | 1"      | = 25.4 mm    |
| Foot              | 1'      | = 0.305 m    |
| Yard              | 1 Yd    | = 0.914 m    |
| Nautical mile     | 1 NM    | = 1852 m     |
| Geographical mile | 1       | = 1855.4 m   |

## Units of area

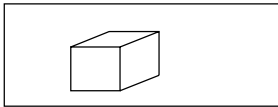


|                   |                   |                             |
|-------------------|-------------------|-----------------------------|
| Square millimetre | 1 mm <sup>2</sup> |                             |
| Square centimetre | 1 cm <sup>2</sup> | = 100 mm <sup>2</sup>       |
| Square decimetre  | 1 dm <sup>2</sup> | = 100 cm <sup>2</sup>       |
| Square metre      | 1 m <sup>2</sup>  | = 100 dm <sup>2</sup>       |
| Are               | 1 a               | = 100 m <sup>2</sup>        |
| Hectare           | 1 ha              | = 100 a                     |
| Square kilometre  | 1 km <sup>2</sup> | = 100 ha                    |
| Square inch       | 1 sq.in           | = 6.45 cm <sup>2</sup>      |
| Square foot       | 1 sq.ft           | = 0.093 m <sup>2</sup>      |
| Square yard       | 1 sq.yd           | = 0.84 m <sup>2</sup>       |
| Square metre      | 1 m <sup>2</sup>  | = 10.76 ft <sup>2</sup>     |
| Acre              | 1                 | = 40.5 a                    |
| 1 Acre            | = 100 cent        | 1 Hectare = 2.47 acres      |
| 1 Cent            | = 436 Sq. ft.     | 1 acre = 0.4047 Hectare     |
| 1 Ground          | = 2400 Sq.ft.     | 1 Hectare = 10000 sq. metre |

## Units of weight



|                   |       |             |
|-------------------|-------|-------------|
| Milligram - force | 1 mgf |             |
| Gram-force        | 1 gf  | = 1000 mgf  |
| Kilogram-force    | 1 kgf | = 1000 gf   |
| Tonne             | 1 t   | = 1000 kgf  |
| Ounce             | 1     | = 28.35 gf  |
| Pound             | 1 lbs | = 0.454 kgf |
| Long ton          | 1     | = 1016 kgf  |
| Short ton         | 1     | = 907 kgf   |

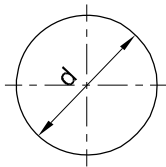


**UNITS OF VOLUME AND CAPACITY**

**Units of volume and capacity**

|                  |                      |                          |
|------------------|----------------------|--------------------------|
| Cubic millimetre | 1 mm <sup>3</sup>    |                          |
| Cubic centimetre | 1 cm <sup>3</sup>    | = 1000 mm <sup>3</sup>   |
| Cubic decimetre  | 1 dm <sup>3</sup>    | = 1000 cm <sup>3</sup>   |
| Cubic metre      | 1 m <sup>3</sup>     | = 1000 dm <sup>3</sup>   |
| Litre            | 1 l                  | = 1 dm <sup>3</sup>      |
| Hectolitre       | 1 hl                 | = 100 l                  |
| Cubic inch       | 1 cu. in             | = 16.387 cm <sup>3</sup> |
| Cubic foot       | 1 cu. ft             | = 28317 cm <sup>3</sup>  |
| Gallon (British) | 1 gal                | = 4.54 l                 |
| 1 cubic metre    | 1 m <sup>3</sup>     | = 1000 litres            |
| 1000 Cu.cm       | 1000 cm <sup>3</sup> | = 1 litre                |
| 1 cubic foot     | 1 ft <sup>3</sup>    | = 6.25 Gallon            |
| 1 litre          | 1lt                  | = 0.22 Gallon            |

**Circular unit**



**CIRCULAR OF UNIT**

**Radian**

Relationship between Radian and Degree

$$1 \text{ Radian} = \frac{180^\circ}{\pi}$$

$$180^\circ = \pi \text{ Radian};$$

$$1 \text{ Degree} = \frac{\pi}{180} \text{ Radian}$$

**Work**



**WORK**

|                 |                      |                               |
|-----------------|----------------------|-------------------------------|
| Kilogram-force  | 1 kgfm               | = 9.80665 J                   |
| Metre           | 1 kgfm               | = 9.80665 Ws                  |
| Joule           | 1 J                  | = 1 Nm                        |
| Watt-second     | 1 Ws                 | = 0.102 kgfm                  |
| Kilowatt hour   | 1 kWh                | = 3.6 x 10 <sup>6</sup> J     |
|                 |                      | = 859.8456 kcal <sub>IT</sub> |
| I.T.Kilocalorie | 1 kcal <sub>IT</sub> | = 426.kgfm                    |

**Power**



**POWER**

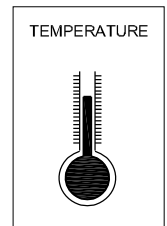
|                             |                                    |
|-----------------------------|------------------------------------|
| Kilogram-force metre/second |                                    |
| 1 kgfm/s                    | = 9.80665 W                        |
| Kilowatt                    | 1 kW = 1000 W = 1000 J/s           |
|                             | = 102 kgfm/s (approx.)             |
| Metric horse power          | 1 HP = 75 kgfm/s                   |
|                             | = 0.736 kW                         |
| 1 Calorie                   | = 4.187J                           |
| I.T.Kilocalorie/hour        | = 1 kcal <sub>IT/h</sub> = 1.163 W |

**Pressure**

|            |                             |                         |                       |  |
|------------|-----------------------------|-------------------------|-----------------------|--|
| Pascal     | 1 Pa                        | = 1 N/m <sup>2</sup>    | 1 atm                 | = 101325 Pa                                      |
| Bar        | 1 bar = 10N/cm <sup>2</sup> | = 100000 Pa–Torr        | 1 torr                | = $\frac{101325}{760} \approx 133.32 \text{ pa}$ |
| Atmosphere | 1 atm                       | = 1 kgf/cm <sup>2</sup> | 1 kgf/cm <sup>2</sup> | = 735.6 mm of mercury                            |

**TEMPERATURE**

| Scale           | Freezing point | Boiling point |
|-----------------|----------------|---------------|
| Centigrade (°C) | 0°C            | 100°C         |
| Fahrenheit (°F) | 32°F           | 212°F         |
| Kelvin (K)      | 273K           | 373K          |
| Reaumur (°R)    | 0°R            | 80°R          |



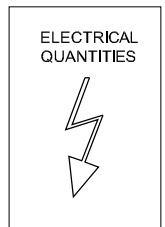
$$\frac{^\circ\text{R}}{80} = \frac{^\circ\text{C}}{100} = \frac{\text{K} - 273}{100} = \frac{^\circ\text{F} - 32}{180}$$

## FORCE

|              |   |
|--------------|---|
| <b>Force</b> | In C.G.S. System : Force (Dyne) = Mass (gm) X Acceleration (cm/sec <sup>2</sup> )     |
|              | In F.P.S. System : Force (Poundal) = Mass (lb) X Acceleration (ft./sec <sup>2</sup> ) |
|              | In M.K.S System : Force (Newton) = Mass (Kg) x Acceleration (mtr./sec <sup>2</sup> )  |
| 1 Dyne       | = 1 gm x 1 cm/sec <sup>2</sup>  |
| 1 Poundal    | = 1 lb x 1 ft/sec <sup>2</sup>  |
| 1 Newton     | = 1 kg x 1 mtr/sec <sup>2</sup> = 10 <sup>5</sup> dynes                               |
| 1gm weight   | = 981 Dynes   |
| 1 lb weight  | = 32 Poundals   |
| 1 kg weight  | = 9.81 Newtons  |

## ELECTRICAL QUANTITIES

|   |                     |                 |           |         |
|---|---------------------|-----------------|-----------|---------|
| V | Electric potential  | V               | Volt      | V(W/A)  |
| E | Electromotive force | V               | Volt      | V(W/A)  |
| I | Electric current    | A               | Ampere    | A       |
| R | Electric resistance | Ω               | Ohm       | Ω (V/A) |
| e | Specific resistance | Ω m             | Ohm metre | Vm/A    |
| G | Conductance         | Ω <sup>-1</sup> | Siemens   | S       |



### Assignment - Answer the following question.

- Convert 320 kilometres into miles \_\_\_\_\_
- Convert 16 tons into kilograms \_\_\_\_\_
- Convert 40 inches into centimetres \_\_\_\_\_
- Convert 8 metres into feet \_\_\_\_\_
- Convert 2.5 gallons into litres \_\_\_\_\_
- Convert 5 litres into gallons \_\_\_\_\_
- 120°C = \_\_\_\_\_ °F.
- Expand the abbreviations of the following
  - N/m<sup>2</sup>
  - RPM
- Convert the following S.I. units as required.
  - Length
    - 3.4 m = \_\_\_\_\_ mm
    - 10.2 km = \_\_\_\_\_ mile
  - Mass
    - 650 g = \_\_\_\_\_ kg
    - 120 mg = \_\_\_\_\_ g
  - Force
    - 1.2 N = \_\_\_\_\_ kg
    - 25 kg = \_\_\_\_\_ N
  - Work, energy, amount of heat
    - 120 KJ = \_\_\_\_\_ J
    - 300 wh = \_\_\_\_\_ kwh
  - Power
    - 0.2 kW = \_\_\_\_\_ W
    - 350 W = \_\_\_\_\_ kW
  - Convert as required.
    - 5 N = \_\_\_\_\_ KN

Unit, Fractions - Factors, HCF, LCM and problems

Prime Numbers and whole Numbers

Factor

A factor is a small number which divides exactly into a bigger number.e.g.

To find the factors of 24, 72, 100 numbers

$$24 = 2 \times 2 \times 2 \times 3$$

$$72 = 2 \times 2 \times 2 \times 3 \times 3$$

$$100 = 2 \times 2 \times 5 \times 5$$

The numbers 2,3,5 are called factors.

Definition of a prime factor

Prime factor is a number which divides a prime number into factors.e.g.

$$57 = 3 \times 19$$

The numbers 3 and 19 are prime factors.

They are called as such, since 3 & 19 also belong to prime number category.

Definition of H.C.F

The Highest Common Factor

The H.C.F of a given group of numbers is the highest number which will exactly divide all the numbers of that group.e.g.

To find the H.C.F of the numbers 24, 72, 100

$$24 = 2 \times 2 \times 2 \times 3$$

$$72 = 2 \times 2 \times 2 \times 3 \times 3$$

$$100 = 2 \times 2 \times 5 \times 5$$

The factors common to all the three numbers are

$$2 \times 2 = 4. \text{ So HCF} = 4.$$

Definition of L.C.M

Lowest common multiple

The lowest common multiple of a group of numbers is the smallest number that will contain each number of the given group without a remainder.e.g.

- Factorise the following numbers

$$7,17,20,66,128$$

7,17 - These two belong to Prime numbers. Hence no factor except unity and itself.

$$\begin{array}{r|l} 2 & 20 \\ 2 & 10 \\ & 5 \end{array}$$

Factors of 20 =  $2 \times 2 \times 5$

$$\begin{array}{r|l} 2 & 66 \\ 3 & 33 \\ & 11 \end{array}$$

Factors of 66 =  $2 \times 3 \times 11$

$$\begin{array}{r|l} 2 & 128 \\ 2 & 64 \\ 2 & 32 \\ 2 & 16 \\ 2 & 8 \\ 2 & 4 \\ & 2 \end{array}$$

Factors of 128 =  $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

- Select prime numbers from 3 to 29  
3,5,7,11,13,17,19,23,29
- Find the HCF of the following group of numbers HCF of 78, 128, 196

$$\begin{array}{r|l} 2 & 78 \\ 3 & 39 \\ & 13 \end{array}$$

$$78 = 2 \times 3 \times 13$$

$$\begin{array}{r|l} 2 & 128 \\ 2 & 64 \\ 2 & 32 \\ 2 & 16 \\ 2 & 8 \\ 2 & 4 \\ & 2 \end{array}$$

$$128 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

$$\begin{array}{r|l} 2 & 196 \\ 2 & 98 \\ & 49 \end{array}$$

$$196 = 2 \times 2 \times 49$$

$$\text{HCF} = 2$$

- Find LCM of 84,92,76

$$\text{LCM} = \begin{array}{r|l} 2 & 84, 92, 76 \\ 2 & 42, 46, 38 \\ 3 & 21, 23, 19 \\ & 7, 23, 19 \end{array}$$

$$\text{LCM} = 2 \times 2 \times 3 \times 7 \times 23 \times 19 = 36708$$

- To find out the LCM of 36, 108, 60

$$\begin{array}{r|l} 2 & 36, 108, 60 \\ 2 & 18, 54, 30 \\ 3 & 9, 27, 15 \\ 3 & 3, 9, 5 \\ & 1, 3, 5 \end{array}$$

LCM of the number

$$36, 108, 60 = 2 \times 2 \times 3 \times 3 \times 3 \times 5 = 540$$

The necessity of finding LCM and HCF arises in subtraction and addition of fractions.

Unit, Fractions - Fractions - Addition, subtraction, multiplication & division

**Description**

A minimal quantity that is not a whole number. For e.g. .

$\frac{1}{5}$  a vulgar fraction consists of a numerator and denominator.

**Numerator/Denominator**

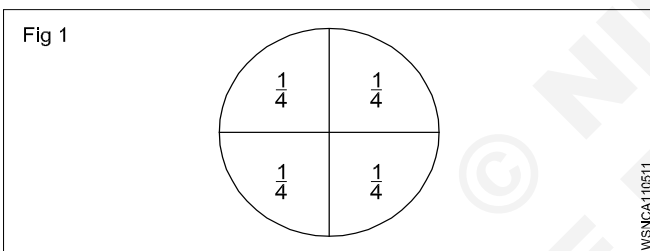
The number above the line in a vulgar fraction showing how many of the parts indicated by the denominator are taken is the numerator. The total number of parts into which the whole quantity is divided and written below the line in a vulgar fraction is the denominator. e.g.

$\frac{1}{4}, \frac{3}{4}, \frac{7}{12}$   
 1,3,7 - numerators                      4,12 - denominators

**Fraction: Concept**

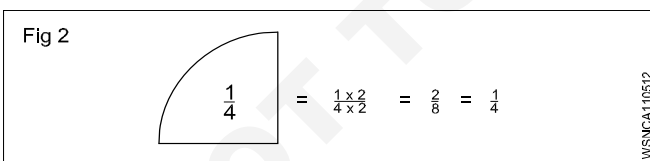
Every number can be represented as a fraction.e.g.

$1\frac{1}{4} = \frac{5}{4}$ , A full number can be represented as an apparent fraction.e.g. (Fig 1)



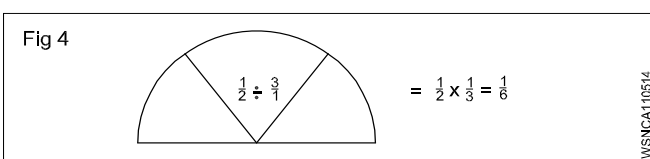
**Fraction: Value**

The value of a fraction remains the same if the numerator and denominator of the fraction are multiplied or divided by the same number.(Fig 2)



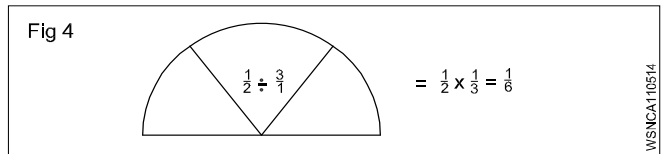
**Multiplication**

When fractions are to be multiplied, multiply all the numerators to get the numerator of the product and multiply all the denominators to form the denominator of the product. (Fig 3)



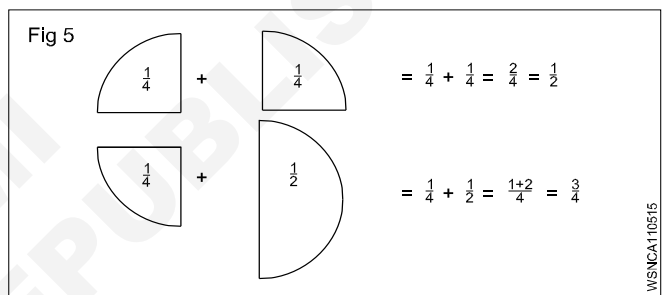
**Division**

When a fraction is divided by another fraction the dividend is multiplied by the reciprocal of the divisor. (Fig 4)



**Addition and Subtraction**

The denominators of the fractions should be the same when adding or subtracting the fractions. Unequal denominators must first be formed into a common denominator. It is the lowest common denominator and it is equal to the product of the most common prime numbers of the denominators of the fractions in question.(Fig 5)



**Examples**

• Multiply  $\frac{3}{4}$  by  $\frac{2}{3}$ ,

$$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12} = \frac{1}{2}$$

• Divide  $\frac{3}{8}$  by  $\frac{3}{4}$ ,

$$\frac{3}{8} \div \frac{3}{4} = \frac{3}{8} \times \frac{4}{3} = \frac{1}{2}$$

• Add  $\frac{3}{4}$  and  $\frac{2}{3}$ ,

$$\frac{3}{4} + \frac{2}{3} = \frac{9}{12} + \frac{8}{12} = \frac{17}{12} = 1\frac{5}{12}$$

• sub  $\frac{7}{16}$  from  $\frac{17}{32}$

$$\frac{17}{32} - \frac{7}{16} = \frac{17}{32} - \frac{14}{32} = \frac{(17 - 14)}{32} = \frac{3}{32}$$

## Types of fractions

- Proper fractions are less than unity. Improper fractions have their numerators greater than the denominators.
- A mixed number has a full number and a fraction.

## Addition of fraction

$$\text{Add } \frac{1}{2} + \frac{1}{8} + \frac{5}{12}$$

To add these fractions we have to find out L.C.M of denominators 2,8,12.

Find L.C.M of 2,8,12

Step 1 L.C.M

|   |        |
|---|--------|
| 2 | 2,8,12 |
| 2 | 1,4,6  |
|   | 1,2,3  |

Factors are 2,2,2,3

Hence L.C.M =  $2 \times 2 \times 2 \times 3 = 24$

Step 2

$$\begin{aligned} \frac{1}{2} + \frac{1}{8} + \frac{5}{12} &= \frac{12}{24} + \frac{3}{24} + \frac{10}{24} \\ &= \frac{12+3+10}{24} = \frac{25}{24} = 1\frac{1}{24} \end{aligned}$$

## Subtraction of fraction

$$\text{subtract } 9\frac{15}{32} \text{ from } 17\frac{9}{16} \text{ or } (17\frac{9}{16} - 9\frac{15}{32})$$

Step 1: Subtract whole number first  $17 - 9 = 8$

Step 2: L.C.M of 16,32 = 32

Since number 16 divides the number 32

$$\text{Subtracting fractions} = \frac{3}{32}$$

Adding with whole number from Step 1

$$\text{we get } 8 + \frac{3}{32} = 8\frac{3}{32}$$

## Common fractions

Problems with plus and minus sign

Example

$$\text{solve } 3\frac{3}{4} + 6\frac{7}{8} - 4\frac{5}{16} - \frac{9}{32}$$

Rule to be followed

- 1 Add all whole numbers
- 2 add all + Numbers
- 3 Add all - Numbers
- 4 Find L.C.M of all denominators

Solution

Step 1: Add whole numbers =  $3 + 6 - 4 = 5$

$$\text{Step 2: Add fractions} = \frac{3}{4} + \frac{7}{8} - \frac{5}{16} - \frac{9}{32}$$

L.C.M of 4,8,16,32 is 32

$$\begin{aligned} &\frac{24 + 28 - 10 - 9}{32} \\ &= \frac{52 - 19}{32} \\ &= \frac{33}{32} = 1\frac{1}{32} \end{aligned}$$

Step 3: Adding again with the whole number

$$\text{we get } 5 + 1\frac{3}{32} = 6\frac{3}{32}$$

## Examples

### Common fractions

- Multiply

$$\text{a } \frac{3}{8} \text{ by } \frac{4}{7} = \frac{3}{8} \times \frac{4}{7} = \frac{3}{14} \quad \text{b } \frac{2}{3} \times \frac{3}{4} \times \frac{5}{8} = \frac{5}{16}$$

- Division

$$\text{a } \frac{5}{16} \div \frac{5}{32} = \frac{5}{16} \times \frac{32}{5} = 2$$

$$\text{b } 4\frac{2}{3} \div 3\frac{1}{7} = \frac{14}{3} \div \frac{22}{7} = \frac{14}{3} \times \frac{7}{22} = \frac{49}{33} = 1\frac{16}{33}$$

- Addition

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$$

L.C.M = 2,4,8 = 8

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{4+2+1}{8} = \frac{7}{8}$$

- Subtraction

$$5\frac{1}{4} - 3\frac{3}{4} = 5 - 3 + \frac{1}{4} - \frac{3}{4}$$

$$= 2 + \frac{1}{4} - \frac{3}{4} = 2 - \frac{2}{4}$$

$$= \frac{9}{4} - \frac{3}{4} = \frac{9-3}{4}$$

$$= \frac{6}{4} = \frac{3}{2} = 1\frac{1}{2}$$

## Assignment

---

1 Convert the following into improper fractions.

a  $1\frac{2}{7} = \underline{\hspace{2cm}}$

b  $4\frac{3}{5} = \underline{\hspace{2cm}}$

c  $3\frac{3}{5} = \underline{\hspace{2cm}}$

2 Convert the following into mixed numbers.

a  $\frac{12}{11} = \underline{\hspace{2cm}}$

b  $\frac{36}{14} = \underline{\hspace{2cm}}$

c  $\frac{18}{10} = \underline{\hspace{2cm}}$

3 Place the missing numbers.

a  $\frac{11}{13} = \frac{x}{91}$  \_\_\_\_\_

b  $\frac{3}{5} = \frac{42}{x}$  \_\_\_\_\_

c  $\frac{9}{14} = \frac{x}{98}$  \_\_\_\_\_

4 Simplify.

a  $\frac{45}{60} = \underline{\hspace{2cm}}$

b  $\frac{8}{12} = \underline{\hspace{2cm}}$

5 Multiply.

a  $5 \times \frac{2}{3} = \underline{\hspace{2cm}}$

b  $\frac{3}{4} \times 2 = \underline{\hspace{2cm}}$

c  $\frac{3}{4} \times \frac{5}{6} = \underline{\hspace{2cm}}$

6 Divide

a  $\frac{1}{4} \div \frac{3}{4} = \underline{\hspace{2cm}}$

b  $6 \div \frac{3}{4} = \underline{\hspace{2cm}}$

c  $\frac{3}{4} \div \frac{2}{7} = \underline{\hspace{2cm}}$

7 Place the missing numbers.

a  $\frac{2}{3} = \frac{1}{12} \times \underline{\hspace{2cm}}$

b  $\frac{14}{24} = \frac{1}{12} \times \underline{\hspace{2cm}}$

c  $\frac{7}{8} = \frac{1}{12} \times \underline{\hspace{2cm}}$

8 Add the followings:

a  $\frac{3}{4} + \frac{7}{12} = \underline{\hspace{2cm}}$

b  $\frac{7}{8} + \frac{3}{4} = \underline{\hspace{2cm}}$

9 Subtract

a  $\frac{4}{5} - \frac{2}{5} = \underline{\hspace{2cm}}$

b  $\frac{5}{6} - \frac{3}{4} = \underline{\hspace{2cm}}$

10 Simplify

a  $2\frac{6}{7} - \frac{3}{8} - \frac{1}{3} - 1\frac{1}{16} = \underline{\hspace{2cm}}$

b  $2\frac{2}{7} - \frac{5}{6} + 8 = \underline{\hspace{2cm}}$

11 Express as improper fractions

a  $5\frac{3}{4}$

b  $3\frac{5}{64}$

c  $1\frac{5}{12}$

**Unit, Fractions - Decimal fractions - Addition, subtraction, multiplication & division**

**Description**

Decimal fraction is a fraction whose denominator is 10 or powers of 10 or multiples of 10 (i.e.) 10, 100, 1000, 10000 etc. Meaning of a decimal number:-

12.3256 means

$$(1 \times 10) + (2 \times 1) + \frac{3}{10} + \frac{2}{100} + \frac{5}{1000} + \frac{6}{10000}$$

**Representation**

The denominator is omitted. A decimal point is placed at different positions of the number corresponding to the magnitude of the denominator

Ex.  $\frac{5}{10} = 0.5, \frac{35}{100} = 0.35, \frac{127}{10000} = 0.0127, \frac{3648}{1000} = 3.648$

**Addition and subtraction**

Arrange the decimal fractions in a vertical order, placing the decimal point of each fraction to be added or subtracted, in succession one below the other, so that all the decimal points are arranged in a straight line. Add or subtract as you would do for a whole number and place the decimal point in the answer below the column of decimal points.

Decimal fractions less than 1 are written with a zero before the decimal point. Example:  $45/100 = 0.45$  (and not simply .45)

Add  $0.375 + 3.686$

$$\begin{array}{r} 0.375 \\ 3.686 \\ \hline 4.061 \end{array}$$

Subtract 18.72 from 22.61

$$\begin{array}{r} 22.61 \\ 18.72 \\ \hline 3.89 \end{array}$$

**Multiplication**

Ignore the decimal points and multiply as whole numbers. Find the total number of digits to the right of the decimal point. Insert the decimal point in the answer such that the number of digits to the right of the decimal point equals to the sum of the digits found to the right of the decimal points in the problem.

Multiply 2.5 by 1.25

=  $25 \times 125 = 3125$ . The sum of the figures to the right of decimal point is 3. Hence the answer is 3.125.

**Division**

Move the decimal point of the divisor to the right to make it a full number. Move the decimal point in the dividend to

the same number of places, adding zeroes if necessary. Then divide.

Divide 0.75 by 0.25

$$0.25 \overline{)0.75}$$

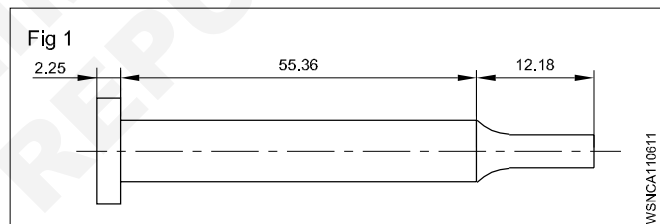
$$\frac{0.75}{0.25} \times \frac{100}{100} = \frac{75}{25}$$

$$25 \overline{)75} = 3$$

**Move the decimal point in the multiplicand to the right to one place if the multiplier is 10, and to two places if the multiplier is 100 and so on. When dividing by 10 move the decimal point one place to the left, and, if it is by 100, move them point by two places and so on.**

**Example**

Allowance allowing 3 mm for cutting off each pin, how many pins can be made from a 900 mm long bar and how much material will be left out?



Total Length of pin =  $2.25 + 55.36 + 12.18$   
= 69.79 mm

Cutting allowance = 3 mm

Total Length = length of pin + cutting allowance  
=  $69.79 \text{ mm} + 3 \text{ mm}$   
= 72.79 mm

Length of the bar = 900 mm

No. of pins to be cut =  $\frac{900}{72.79} = 12.394$   
= 12 pins

Left out material = Total length - length of pin + cutting allowance  
=  $900 - 12 \times 69.79 + 12 \times 3$   
=  $900 - 837.48 + 36$   
=  $900 - 873.48$

Left out length = 26.52 mm



### Conversion of Decimals into fractions and vice-versa

- Convert decimal into fractions

#### Example

Convert 0.375 to a fraction

Now place 1 under the decimal point followed by as many zeros as there are numbers

$$0.375 = \frac{375}{1000} = \frac{15}{40} = \frac{3}{8}$$

$$0.375 = \frac{3}{8}$$

- Convert fraction into decimal

#### Example

- Convert  $\frac{9}{16}$  to a decimal

Proceed to divide  $\frac{9}{16}$  in the normal way of division but put zeros (as required) after the number 9 (Numerator)

$$\begin{array}{r} 0.5625 \\ 16 \overline{)90000} \\ \underline{80} \phantom{00} \\ 100 \phantom{0} \\ \underline{96} \phantom{0} \\ 40 \phantom{0} \\ \underline{32} \phantom{0} \\ 80 \phantom{0} \\ \underline{80} \phantom{0} \\ 0 \end{array}$$

$$\frac{9}{16} = 0.5625$$

### Recurring decimals

While converting from fraction to decimals, some fractions can be divided exactly into a decimal. In some fractions the quotient will not stop. It will continue and keep recurring. These are called recurring decimals.

#### Examples

- convert  $\frac{1}{3}, \frac{2}{3}, \frac{1}{7}$

a  $\frac{1}{3} = \frac{10000}{3} = 0.3333$  – Recurring

b  $\frac{2}{3} = \frac{20000}{3} = 0.666$  – Recurring

c  $\left( \frac{1}{7} = \frac{10000}{7} = 0.142857142 \right)$  – Recurring

### Method of writing approximations in decimals

$$1.73556 = 1.7356 \text{ Correct to 4 decimal places}$$

$$5.7343 = 5.734 \text{ Correct to 3 decimal places}$$

$$0.9345 = 0.94 \text{ Correct to 2 decimal places}$$

### Multiplication and division by 10, 100, 1000

Multiplying decimals by 10

A decimal fraction can be multiplied by 10, 100, 1000 and so on by moving the decimal point to the right by as many places as there are zeros in the multiplier.

- $4.645 \times 10 = 46.45$  (one place)
- $4.645 \times 100 = 464.5$  (two places)
- $4.645 \times 1000 = 4645$  (three places)

Dividing decimals by 10

A decimal fraction can be divided by 10, 100, 1000 and so on, by moving the decimal point to the left by as many places as required in the divisor by putting zeros

#### Examples

- $3.732 \div 10 = 0.3732$  (one place)
- $3.732 \div 100 = 0.03732$  (two places)
- $3.732 \div 1000 = 0.003732$  (three places)

#### Examples

- Rewrite the following number as a fraction

453.273

$$= (4 \times 100) + (5 \times 10) + (3 \times 1) + \frac{2}{10} + \frac{7}{100} + \frac{3}{1000}$$

$$= 453 \frac{273}{1000}$$

- Write the representation of decimal places in the given number 0.386

- 3 - 1st decimal place
- 8 - 2nd decimal place
- 6 - 3rd decimal place

- Write approximations in the following decimals to 3 places.

- a  $6.9453 \longrightarrow 6.945$
- b  $8.7456 \longrightarrow 8.746$

- Convert fraction to decimal

$$\frac{21}{24} = \frac{7}{8} = 0.875$$

- Convert decimal to fraction

$$0.0625 = \frac{625}{10000} = \frac{5}{80} = \frac{1}{16}$$

## Assignment

---

1 Write down the following decimal numbers in the expanded form.

a 514.726

b 902.524

2 Write the following decimal numbers from the expansion.

a  $500 + 70 + 5 + \frac{3}{10} + \frac{2}{100} + \frac{9}{1000}$

b  $200 + 9 + \frac{1}{10} + \frac{3}{100} + \frac{5}{1000}$

3 Convert the following decimals into fractions in the simplest form.

a 0.72

b 5.45

c 3.64

d 2.05

4 Convert the following fraction into decimals

a  $\frac{3}{5}$

b  $\frac{10}{4}$

c  $24 \frac{54}{1000}$

d  $\frac{12}{25}$

e  $\frac{8}{25}$

f  $1 \frac{3}{25}$

5 Addition of decimals

a  $4.56 + 32.075 + 256.6245 + 15.0358$

b  $462.492 + 725.526 + 309.345 + 626.602$

6 Subtract the following decimals

a  $612.5200 - 9.6479$

b  $573.9246 - 215.6000$

7 Add and subtract the following

a  $56.725 + 48.258 - 32.564$

b  $16.45 + 124.56 + 62.7 - 3.243$

8 Multiply the following

a By 10,100,1000

i  $3.754 \times 10$

ii  $8.964 \times 100$

iii  $2.3786 \times 1000$

iv  $0.005 \times 1000$

b By whole numbers

i  $8.4 \times 7$

ii  $56.72 \times 8$

c By another decimal figure (use calculator)

i  $15.64 \times 7.68$

ii  $2.642 \times 1.562$

9 Divide the following

a  $\frac{62.5}{25}$

b  $\frac{64.56}{10}$

c  $\frac{0.42}{100}$

d  $\frac{48.356}{1000}$

10 Division

a  $\frac{16.8}{1.2}$

b  $\frac{1.54}{1.1}$

11 Change the fraction into a decimal

i  $1 \frac{5}{8}$

ii  $\frac{12}{25}$

12 Find the value

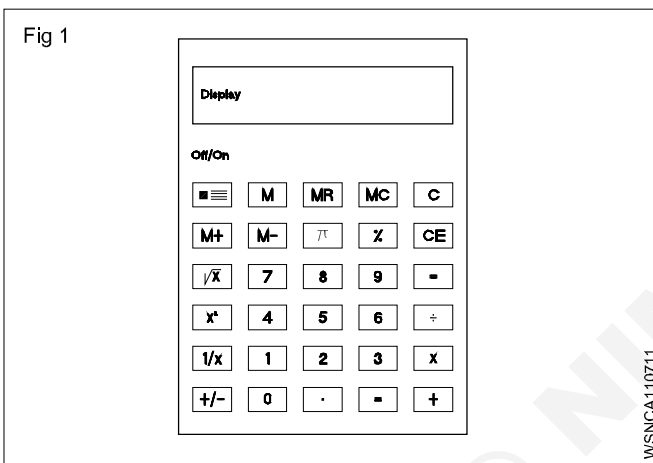
$20.5 \times 40 \div 10.25 + 18.50$

Unit, Fractions - Solving problems by using calculator

A pocket calculator allows to spend less time in doing tedious calculations. A simple pocket calculator enables to do the arithmetical calculations of addition, subtraction, multiplication and division, while a scientific type of calculator can be used for scientific and technical calculations also.

No special training is required to use a calculator. But it is suggested that a careful study of the operation manual of the type of the calculator is essential to become familiar with its capabilities. A calculator does not think and do. It is left to the operator to understand the problem, interpret the information and key it into the calculator correctly.

Constructional Details (Fig 1)



The key board is divided into five clear and easily recognizable areas and the display.

• Data entry keys

The entry keys are from 0 .....to 9

and a key for the decimal point . .

• Clearing keys

These keys have the letter 'C'

C CLR Clear totally

CE Clear entry only

CM , MC Clear memory

- + Addition key
- Subtraction key
- x Multiplication key
- ÷ Division key
- = Equals key to display the result

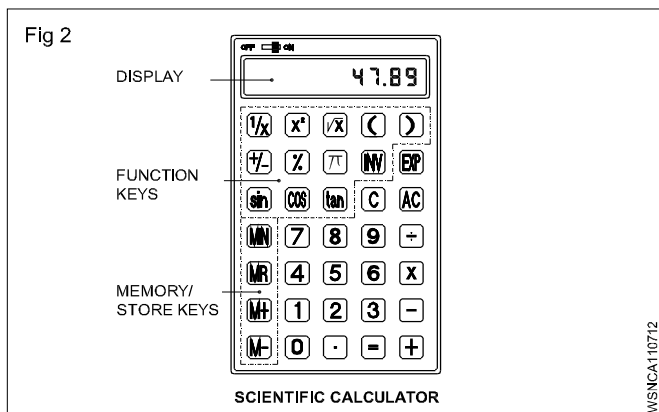
• Function keys

- π Pi key
- $\sqrt{x}$  Square root key
- % Percentage key
- +/- Sign change key
- $x^2$  Square key
- $\frac{1}{x}$  Reciprocal key

• Memory keys

- M Store the display number
- M+ The displayed value is added to the memory
- M- The displayed value is subtracted from the memory
- MR RCL The stored value is recalled on to the display

Further functional keys included in Scientific calculators are as shown in Fig 2.



$\sin$   $\cos$   $\tan$   $($   $)$  For trigonometric functions and for brackets

$\text{Exp}$  Exponent key

$\text{INV}$  Some of the keys have coloured lettering above or below them. To use a function in coloured lettering, press INV key. INV will appear on the display. Then press the key that the coloured lettering identifies. INV will disappear from the display.

$\log$  ,  $\text{INV}$   $10^x$  to obtain the logarithm of the displayed number and the antilogarithm of the displayed value.

$\text{INV}$   $\text{R-P}$  to convert displayed rectangular coordinates into polar coordinates.

$\text{INV}$   $\text{P-R}$  to convert displayed polar coordinates into rectangular coordinates.

#### • The display

The display shows the input data, interim results and answers to the calculations.

**The arrangement of the areas can differ from one make to another. Keying in of the numbers is done via an internationally agreed upon set of ten keys in the order that the numbers are written.**

Rules and Examples:

- **Addition:** Example  $18.2 + 5.7$

| Sequence   | Input  | Display |
|--|--------|---------|
| Input of the 1st term of the sum                                 | $18.2$ | 18.2    |
| Press + key  | $+$    | 18.2    |
| Input 2nd term of the sum. the first term goes into the register | $5.7$  | 5.7     |
| Press the = key  | $=$    | 23.9    |

- **Subtraction:** Example  $128.8 - 92.9$

| Sequence   | Input   | Display |
|--|---------|---------|
| Enter the subtrahend                                     | $128.8$ | 128.8   |
| Press - key  | $-$     | 128.8   |
| Enter the minuend. The subtrahend goes into the register | $92.9$  | 92.9    |
| Press the = key  | $=$     | 35.9    |

- **Multiplication:** Example  $0.47 \times 2.47$

| Sequence  | Input    | Display |
|---|----------|---------|
| Enter multiplicand                              | $0.47$   | 0.47    |
| Press x key                                     | $\times$ | 0.47    |
| Enter multiplier, multiplicand goes to register | $2.47$   | 2.47    |
| Press = key                                     | $=$      | 1.1609  |

- **Division:** Example  $18.5/2.5$

| Sequence                               | Input  | Display |
|--|--------|---------|
| Enter the dividend                     | $18.5$ | 18.5    |
| Press ÷ Key                            | $\div$ | 18.5    |
| Enter the divisor goes to the register | $2.5$  | 2.5     |
| Press = key                            | $=$    | 7.4     |

- **Multiplication & Division:**  
Example :  $2.5 \times 7.2 / 4.8 \times 1.25$

| Sequence            | Input   | Display |
|---------------------|---------|---------|
| Enter 2.5           | 2 . 5   | 2.5     |
| Press x key         | x       | 2.5     |
| Enter 7.2           | 7 . 2   | 7.2     |
| Press ÷ key         | ÷       | 18      |
| Enter Open bracket  | (       |         |
| Enter 4.8           | 4 . 8   | 4.8     |
| Press x key         | x       | 4.8     |
| Enter 1.25          | 1 . 2 5 | 1.25    |
| Enter Close bracket | )       | 6       |
| Press = key         | =       | 3.0     |

- Store in memory Example  $(2+6) (4+3)$

| Sequence                      | Input     | Display |
|-------------------------------|-----------|---------|
| Workout for the first bracket | 2         | 2       |
|                               | +         | 2       |
|                               | 6         | 6       |
|                               | =         | 8       |
| Store the first result in     | STO , M   | 8       |
| x                             | or M+     |         |
| Workout for the 2nd bracket   | 4         | 4       |
|                               | +         | 4       |
|                               | 3         | 3       |
|                               | =         | 7       |
| Press x key                   | x         | 7       |
| Recall memory                 | RCL or MR | 8       |
| Press = key                   | =         | 56      |

- **Percentage:** Example 12% of 1500

| Sequence    | Input   | Display |
|-------------|---------|---------|
| Enter 1500  | 1 5 0 0 | 1500    |
| Press x key | x       | 1500    |
| Enter 12    | 1 2     | 12      |
| Press INV % | INV %   | 12      |
| Press = key | =       | 180     |

- **Square root:** Example  $\sqrt{2} + \sqrt{3 \times 5}$

| Sequence                | Input  | Display   |
|-------------------------|--|-----------|
| Enter 2                 | 2  | 2         |
| Press $\sqrt{a}$ key    | $\sqrt{a}$                                       | 1.414     |
| Press + key             | +  | 1.414     |
| Press bracket key       | (  | 1.414     |
| Enter 3                 | 3  | 3         |
| Press $\sqrt{a}$ key    | $\sqrt{a}$                                       | 1.732     |
| Press x key             | x  | 1.732     |
| Enter 5                 | 5  | 5         |
| Press $\sqrt{a}$ key    | $\sqrt{a}$                                       | 2.236     |
| Press bracket close key | )  | 3.873     |
| Press = key             | =  | 5.2871969 |
|                         | 2 $\sqrt{a}$ + ( 3 $\sqrt{a}$ x 5 $\sqrt{a}$ ) = | 5.2871969 |

$$\sqrt{2} + \sqrt{3 \times 5} = 5.287$$

- **Common logarithm:** Example  $\log 1.23$

| Sequence | Input         | Display   |
|----------|---------------|-----------|
|          | 1 . 2 3 log = | 0.0899051 |

- **Power:** Example  $123 + 30^2$

| Sequence | Input                            | Display |
|----------|----------------------------------|---------|
|          | 1 2 3 + 3 0 INV X <sup>2</sup> = | 1023    |

- Before starting the calculations be sure to press the 'ON' key and confirm that '0' is shown on the display.
- Do not touch the inside portion of the calculator. Avoid hard knocks and unduly hard pressing of the keys.
- Maintain and use the calculator in between the two extreme temperatures of 0° and 40° C.

- Never use volatile fluids such as lacquer, thinner, benzene while cleaning the unit.
- Take special care not to damage the unit by bending or dropping.
- Do not carry the calculator in your hip pocket.

## Assignment

1 Using calculator solve the following

- $625 + 3467 + 20 + 341 + 6278 =$  \_\_\_\_\_
- $367.4 + 805 + 0.7 + 7.86 + 13.49 =$  \_\_\_\_\_
- $0.043 + 1.065 + 13.0 + 34.76 + 42.1 =$  \_\_\_\_\_
- $47160 + 1368.4 + 0.1 + 1.6901 + 134.267 =$  \_\_\_\_\_

2 Using calculator simplify the following

- $24367 - 4385 =$  \_\_\_\_\_
- $9.643 - 0.7983 =$  \_\_\_\_\_
- $4382.01 - 381.3401 =$  \_\_\_\_\_
- $693.42 - 0.0254 =$  \_\_\_\_\_

3 Using calculator find the values of the following

- $23 \times 87 =$  \_\_\_\_\_
- $1376 \times 0.81 =$  \_\_\_\_\_
- $678 \times 243 =$  \_\_\_\_\_
- $0.75 \times 0.24 =$  \_\_\_\_\_

4 Using calculator solve the following

- $22434 \div 3 =$  \_\_\_\_\_
- $4131 \div 243 =$  \_\_\_\_\_
- $469890 \div 230 =$  \_\_\_\_\_
- $3.026 \div 0.89 =$  \_\_\_\_\_

5 Solve the following

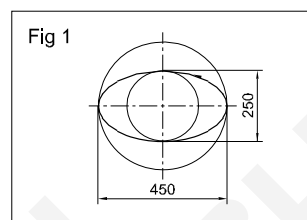
- $\frac{1170 \times 537.5}{13 \times 215} =$  \_\_\_\_\_
- $\frac{28.2 \times 18 \times 3500}{1000 \times 3 \times 0.8} =$  \_\_\_\_\_

6 Solve the following

- $\frac{(634 + 128) \times (384 - 0.52)}{8 \times 0.3} =$  \_\_\_\_\_

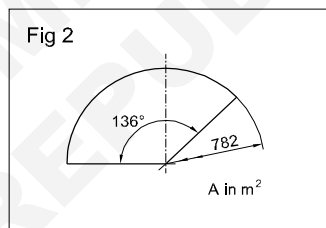
$$b \frac{(389 - 12.2) \times (842 - 0.05 - 2.6)}{(3.89 - 0.021) \times (28.1 + 17.04)} =$$

7



2a = 450 mm (major axis)  
2b = 250 mm (minor axis)  
Perimeter of the ellipse  
A = \_\_\_\_\_ metre<sup>2</sup>  
Hint A =  $\pi \times a \times b$  unit<sup>2</sup>

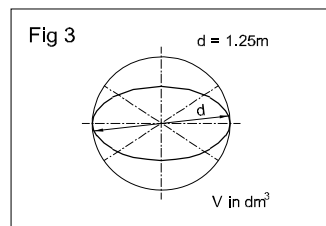
8



$\phi = 782$  mm  
 $\alpha = 136^\circ$   
Area of the sector  
A = \_\_\_\_\_

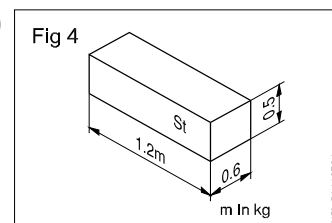
$$\text{Hint } A = \frac{\pi \times d^2}{4} \times \frac{\alpha}{360^\circ}$$

9



d = 1.25 metre  
V = \_\_\_\_\_ dm<sup>3</sup>  
Volume of sphere  
Hint V =  $\frac{4}{3} \pi r^3$

10



L = 1.2 metres  
B = 0.6 metre  
H = 0.5 metre  
' $\rho$ ' (rho) density of steel  
= 7.85 kg/dm<sup>3</sup>  
m = \_\_\_\_\_ kg  
(mass 'm = V x  $\rho$ )

Square root, Ratio and Proportions, Percentage - Square and square root

- a basic number
- 2 exponent

√ radial sign indicating the square root.

√a<sup>2</sup> square root of 'a' square

a<sup>2</sup> radicand

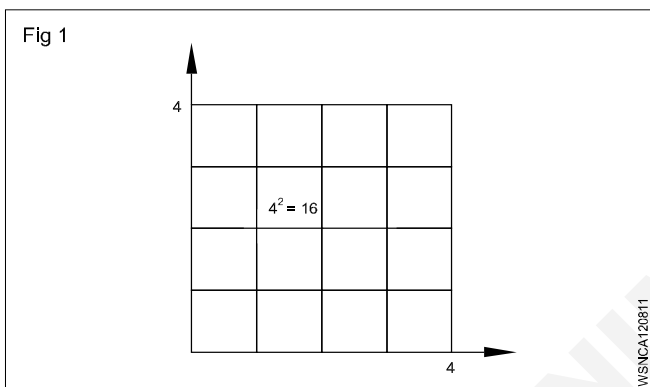
**Square number**

The square of a number is the number multiplied by itself.

Basic number x basic number = Square number

a x a = a<sup>2</sup>

4 x 4 = 4<sup>2</sup> = 16



**Splitting up**

A square area can be split up into sub-areas. The largest square of 36 is made up of a large square 16, a small square 4 and two rectangles 8 each.

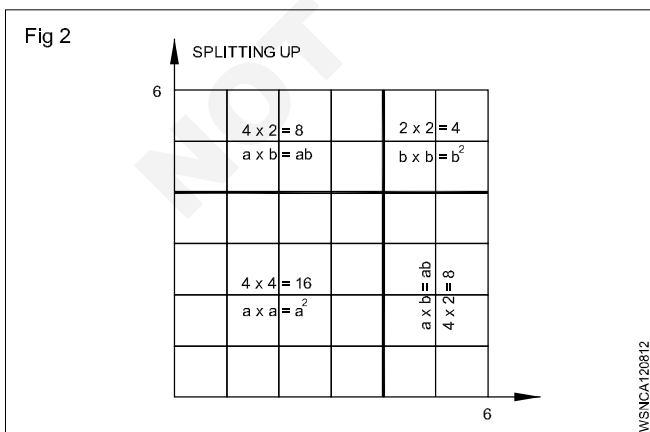
Large square 4 x 4 = 16                      a<sup>2</sup>

Two rectangles 2 x 4 x 2 = 16              2ab

Small square 2 x 2 = 4                      b<sup>2</sup>

Sum of sub-areas = 36 = a<sup>2</sup> + 2ab + b<sup>2</sup>

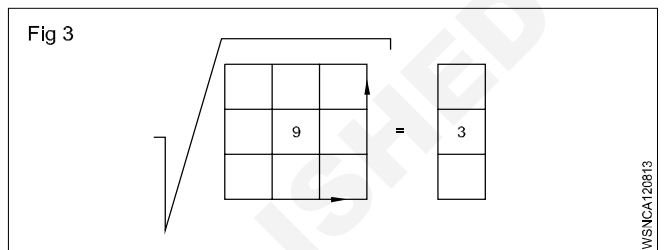
√36 = √a<sup>2</sup> + 2ab + b<sup>2</sup>



Result: In order to find the square root, we split up the square numbers.

**Extracting the square root procedure**

- Starting from the decimal point form groups of two figures towards right and left. Indicate by a prime symbol. √4624.00
- Find the root of the first group, calculate the difference, bring down the next group.
- Multiply the root by 2 and divide the partial radicand.
- Enter the number thus calculated in the divisor for the multiplication.



If there is a remainder, repeat the procedure.

|     |       |
|-----|-------|
| 68  | 46,24 |
| 6   | 36    |
| 128 | 1024  |
|     | 1024  |
|     | 0     |

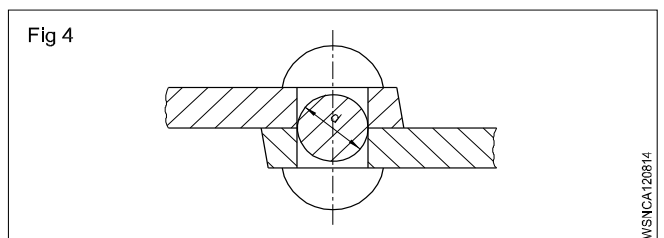
√4624 = 68

**Basic number x basic number = Square**

√Square number = basic number

**Example**

The cross-section of a rivet is 3.46 cm<sup>2</sup>. Calculate the diameter of the hole.



Rivet cross-section is the hole cross-section.

To find 'd',

Given that Area = 3.46 cm<sup>2</sup>

Area = 0.785 x d<sup>2</sup> (formula)

3.46 cm<sup>2</sup> = d<sup>2</sup> x 0.785

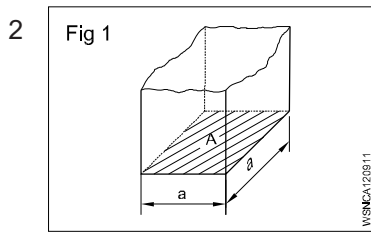
$$d^2 = \frac{3.46 \text{ cm}^2}{0.785}$$

$$d = \sqrt{\frac{3.46}{0.785}} \text{ cm}$$

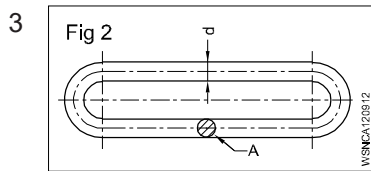
d = 2.1 cm (or) 21mm

Square root, Ratio and Proportions, Percentage - Simple problems using calculator

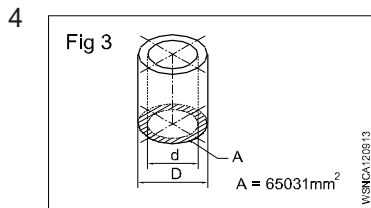
- 1 a  $\sqrt{2916} =$  \_\_\_\_\_.
- b  $\sqrt{45796} =$  \_\_\_\_\_.
- c  $\sqrt{8.2944} =$  \_\_\_\_\_.
- d  $\sqrt{63.845} =$  \_\_\_\_\_.



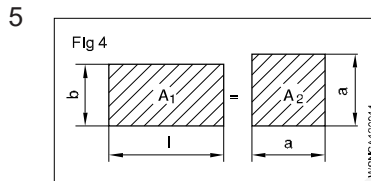
A = 2025 mm<sup>2</sup>  
a = \_\_\_\_\_ mm



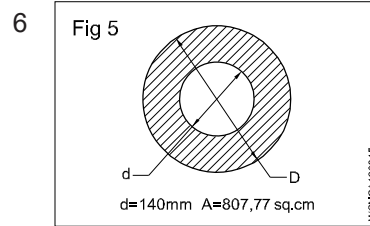
A = 176.715 mm<sup>2</sup>  
d = \_\_\_\_\_ mm



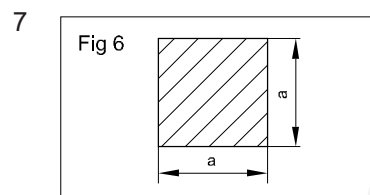
A = 65031 mm<sup>2</sup>  
d = 140 mm  
D = \_\_\_\_\_ mm



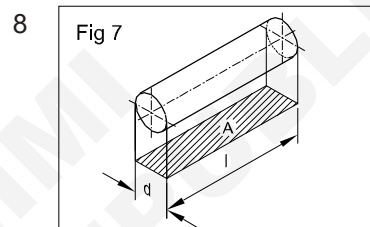
l = 58 cm  
b = 45 cm  
A<sub>1</sub> = A<sub>2</sub>  
a = \_\_\_\_\_ cm



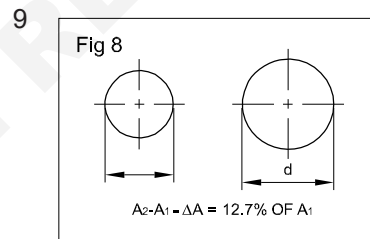
A = 807.77 cm<sup>2</sup>  
d = 140 mm  
D = \_\_\_\_\_ mm



a x a = 543169 mm<sup>2</sup>  
a = \_\_\_\_\_ mm



d : l = 1:1.5  
A = 73.5 mm<sup>2</sup>  
d = \_\_\_\_\_ mm



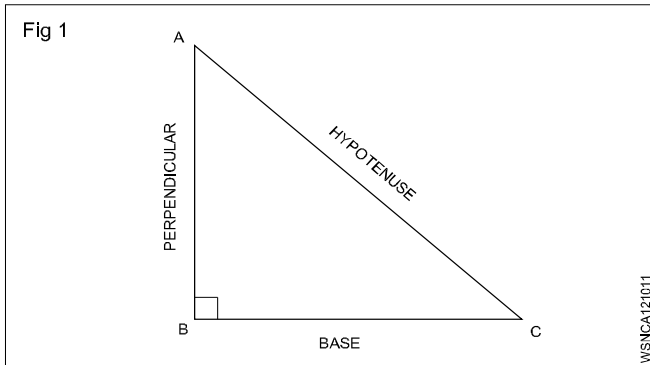
increase in area  
A = 12.7%  
A = 360 mm<sup>2</sup>  
d = \_\_\_\_\_ mm  
(d = diameter after the increase in area)



Square root, Ratio and Proportions, Percentage - Applications of pythagoras theorem and related problems

Applications of Pythagoras Theorem

Some of the applications of the Pythagoras theorem are; (Fig 1)



- 1 The Pythagoras theorem is commonly used to find the lengths of sides of a right-angled triangle.
- 2 It is used to find the length of the diagonal of a square.
- 3 Pythagoras theorem is used in trigonometry to find the trigonometric ratios like sin, cos, tan, cosec, sec and cot.
- 4 Pythagoras theorem is used in security cameras for face recognition.
- 5 Architects use the technique of the Pythagoras theorem for engineering and construction fields.
- 6 The Pythagoras theorem is applied in surveying the mountains.
- 7 It is also used in navigation to find the shortest route.
- 8 By using the Pythagoras theorem, we can derive the formula for base, perpendicular and hypotenuse.
- 9 Painters use ladders to paint on high buildings with the help of the Pythagoras theorem.
- 10 Pythagoras theorem is used to calculate the steepness of slopes of hills or mountains.
- 11 The converse of the Pythagoras theorem is used to check whether a triangle is a right triangle or not.

Application of pythagoras theorem in real life

Pythagoras theorem states that

"In a right-angled triangle, the square of the hypotenuse side is equal to the sum of squares of the other two sides".

- 1 The sides of this triangle have been named Perpendicular, Base and Hypotenuse.
- 2 The hypotenuse is the longest side, as it is opposite to the angle 90°.

- 3 The sides of a right triangle (say AB, BC and CA) which have positive integer values, when squared, are put into an equation, also called a Pythagorean triplet.
- 4 To calculate the length of staircase required to reach a window
- 5 To find the length of the longest item can be kept in your room.
- 6 To find the steepness of the hills or mountains.
- 7 To find the original height of a tree broken due to heavy rain and lying on itself
- 8 To determine heights and measurements in the construction sites.

Examples

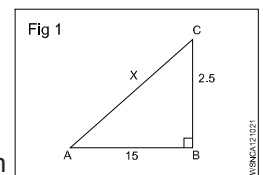
- 1 What is the side AC if AB = 15 cm, BC = 25 cm.

$$AC^2 = AB^2 + BC^2$$

$$= 15^2 + 25^2$$

$$= 225 + 625 = 850$$

$$AC = \sqrt{850} = 29.155 \text{ cm}$$



- 2 What is the side BC if AB = 10 cm, AC = 30 cm.

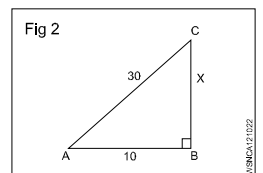
$$AC^2 = AB^2 + BC^2$$

$$30^2 = 10^2 + BC^2$$

$$900 = 100 + BC^2$$

$$BC^2 = 900 - 100 = 800$$

$$BC = 28.284 \text{ cm}$$



- 3 What is the side AB if BC = 20 cm, AC = 35 cm.

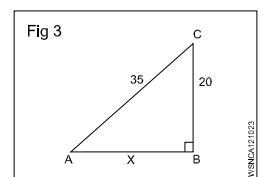
$$AC^2 = AB^2 + BC^2$$

$$35^2 = AB^2 + 20^2$$

$$1225 = AB^2 + 400$$

$$AB^2 = 1225 - 400 = 825$$

$$AB = 28.72 \text{ cm}$$



- 4 What is the value of side BC if AB = 8 cm, AC = 24 cm.

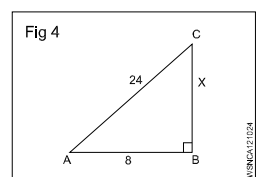
$$AC^2 = AB^2 + BC^2$$

$$24^2 = 8^2 + BC^2$$

$$576 = 64 + BC^2$$

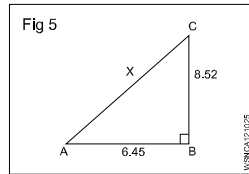
$$BC^2 = 576 - 64 = 512$$

$$BC = \sqrt{512} = 22.63 \text{ cm}$$



- 5 What is the value side AC if AB = 6.45 cm, BC = 8.52 cm.

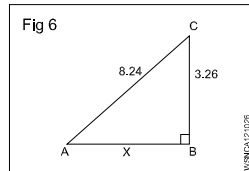
$$\begin{aligned} AC^2 &= AB^2 + BC^2 \\ AC^2 &= 6.45^2 + 8.52^2 \\ AC^2 &= 41.60 + 72.59 \\ &= 114.19 \end{aligned}$$



$$AC = \sqrt{114.19} = 10.69 \text{ cm}$$

- 6 What is the value of side AB if BC = 3.26 cm, AC = 8.24 cm.

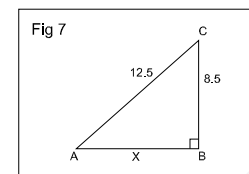
$$\begin{aligned} AC^2 &= AB^2 + BC^2 \\ 8.24^2 &= AB^2 + 3.26^2 \\ 67.9 &= AB^2 + 10.63 \\ AB^2 &= 67.9 - 10.63 \\ &= 57.27 \end{aligned}$$



$$AB = \sqrt{57.27} = 7.57 \text{ cm}$$

- 7 What is the value of side AB if AC = 12.5 cm, BC = 8.5 cm.

$$\begin{aligned} AC^2 &= AB^2 + BC^2 \\ 12.5^2 &= AB^2 + 8.5^2 \\ 156.25 &= AB^2 + 72.25 \\ AB^2 &= 156.25 - 72.25 \\ &= 84 \end{aligned}$$



$$AB = \sqrt{84} = 9.17 \text{ cm}$$

- 8 A ladder of 12.5 metre long is placed with upper end against a wall. The lower end being 7.5 metres from the wall. What height is the upper end above the ground.

$$\begin{aligned} AC^2 &= AB^2 + BC^2 \\ BC^2 &= AC^2 - AB^2 \\ BC^2 &= x^2 \\ AC^2 &= AB^2 + BC^2 \\ 12.5^2 &= x^2 + 7.5^2 \\ x^2 &= (12.5)^2 - (7.5)^2 \\ &= (12.5 + 7.5)(12.5 - 7.5)^2 \\ &= 20 \times 5 \\ &= \sqrt{100} = 10 \end{aligned}$$

$$x = 10 \text{ m}$$

- 9 What is the value of AB.

$$\begin{aligned} AC^2 &= AB^2 + BC^2 \\ AB^2 &= AC^2 - BC^2 \\ AB^2 &= x^2 \\ AC^2 &= AB^2 + BC^2 \\ 10^2 &= x^2 + 6^2 \\ x^2 &= 10^2 - 6^2 \\ &= 100 - 36 = 64 \end{aligned}$$

$$x = \sqrt{64}$$

$$= 8$$

$$AB = 8$$

## Assignment

- 1 What is the value of side AB, in a right angled triangle of side AC = 10 cm and BC = 5 cm.
- 2 What is the value of side AC, in a right angled triangle of side AB = 6.5 cm and BC = 4.5 cm.
- 3 What is the value of side BC, in a right angled triangle of side AC = 14.5 cm and AB = 10.5 cm.
- 4 What is the value of side AC, in a right angled triangle of side AB = 7 cm and BC = 5 cm.
- 5 What is the value of side BC, in a right angled triangle of side AC = 13.25 cm and AB = 8.75 cm.

Square root, Ratio and Proportions, Percentage - Ratio and proportion

Ratio

Introduction

It is the relation between two quantities of the same kind and is expressed as a fraction.

Expression

a, b two quantities of the same kind.  $\frac{a}{b}$  or a:b or a ÷ b or

a in b is the ratio.

Ratio is always reduced to the lowest terms.

Example

$$7:14 = \frac{7}{14} = \frac{1}{2} = 1:2$$

Proportion

It is the equality between the ratios, a : b is a ratio and c : d is another ratio. Both ratios are equal. Then

$$a : b :: c : d \text{ or } \frac{a}{b} = \frac{c}{d}$$

Example

$$250 : 2000 :: 1 : 8$$

Proportion fundamentals

If  $\frac{a}{b} = \frac{c}{d}$  then

- $ad = bc$

- $\frac{a}{c} = \frac{b}{d}$

- $\frac{b}{a} = \frac{d}{c}$

- $\frac{a+b}{b} = \frac{c+d}{c}$  and  $\frac{a+b}{a} = \frac{c+d}{d}$

- $\frac{a-b}{b} = \frac{c-d}{d}$

- $\frac{a+b}{b+d} = \frac{a}{c} = \frac{c}{d}$

$$3:4::6:8 \text{ or } \frac{3}{4} = \frac{6}{8}$$

- $3 \times 8 = 6 \times 4$

- $\frac{3}{6} = \frac{4}{8}$

- $\frac{4}{3} = \frac{8}{6}$

- $\frac{3+4}{4} = \frac{6+8}{8}$

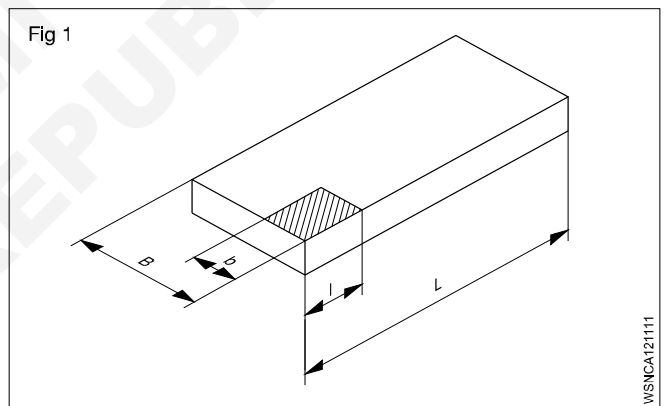
- $\frac{3-4}{4} = \frac{6-8}{8}$

- $\frac{3+6}{4+8} = \frac{9}{12} = \frac{3}{4}$

Ratio - relation of two quantities of the same kind.  
Proportion - equality between two ratios.

Example

- A steel plate of 800 x 1400 mm is to be drawn to a scale of 1:20. What will be the lengths in the Fig 1.

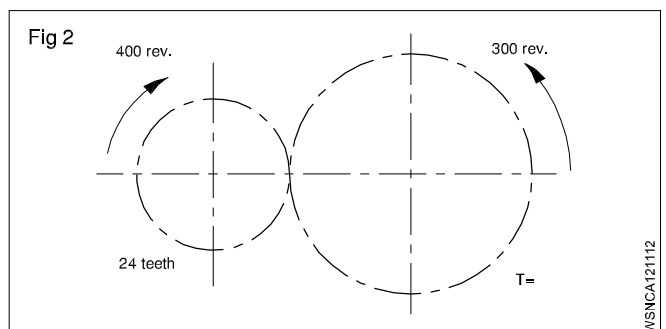


The reduction ratio is  $\frac{1}{20}$ .

B is reduced from 800 to  $800 \times \frac{1}{20} = 40$  mm.

L is reduced from 1400 to  $1400 \times \frac{1}{20} = 70$  mm.

- Find the number of teeth of the larger gear in the gear transmission shown in the Fig 2.



Speed ratio = 400 : 300

Teeth ratio = 24:T

$$\frac{400}{300} = \frac{T}{24}$$

$$\therefore T = \frac{24 \times 400}{300} = 32 \text{ Teeth}$$

Find the ratio of A:B:C

If A:B= 2:3 and B:C=4:5

$$A:B = 2:3$$

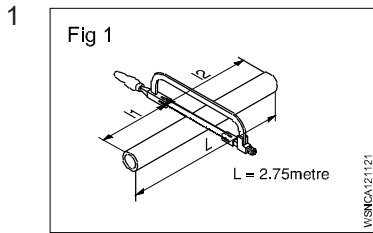
$$B:C = 4:5$$

$$A:B = 8 : 12 \text{ (Ratio 2:3 multiply by 4)}$$

$$B:C = 12:15 \text{ (Ratio 4:5 multiply by 3)}$$

$$\therefore A:B:C = 8:12:15$$

## Assignment

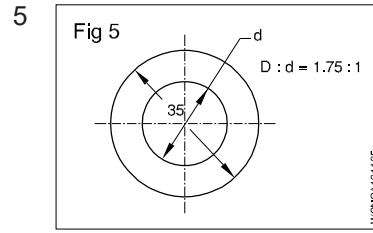


$$l_1 : l_2 = 2:3$$

$$L = 2.75 \text{ metres}$$

$$l_1 = \text{_____ metres}$$

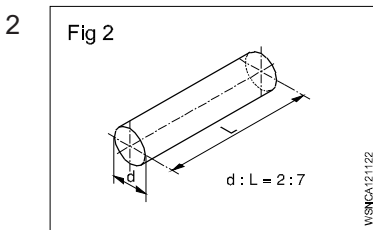
$$l_2 = \text{_____ metres}$$



$$D : d = 1.75 : 1$$

$$D = 35 \text{ mm}$$

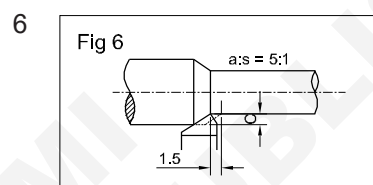
$$d = \text{_____ mm}$$



$$d : L \text{ of shaft} = 2:7$$

$$d = 40 \text{ mm}$$

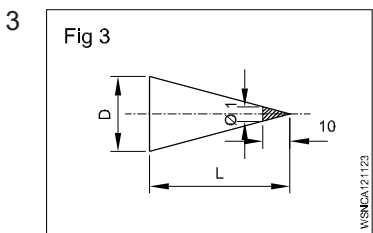
$$L = \text{_____ mm}$$



$$a : s = 5 : 1$$

$$s = 1.5 \text{ mm}$$

$$a = \text{_____ mm}$$



$$D : L = 1 : 10$$

$$L = 150 \text{ mm}$$

$$D = \text{_____ mm}$$

7 A:B=9:12

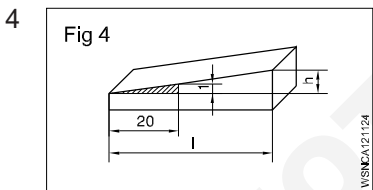
B:C=8:10

Then A:B:C= \_\_\_\_\_

8 A:B=5:6

B:C=3:4

Then A:B:C= \_\_\_\_\_



$$\frac{\Delta h}{l} = \frac{1}{20}$$

$$l = 140 \text{ mm}$$

$$\Delta h = \text{_____ mm}$$

9 A:55=9:11

$$A = \text{_____}$$

10 15:9.3=40:x

$$x = \text{_____}$$

Square root, Ratio and Proportions, Percentage - Ratio and Proportion - Direct and indirect proportions

Proportion

Description

It is the equality between the ratios, a:b is a ratio and c:d is another ratio. Both ratios are equal. Then

$a : b :: c : d$  or e.g.  $250 : 2000 :: 1 : 8$

Rule of three

A three step calculation

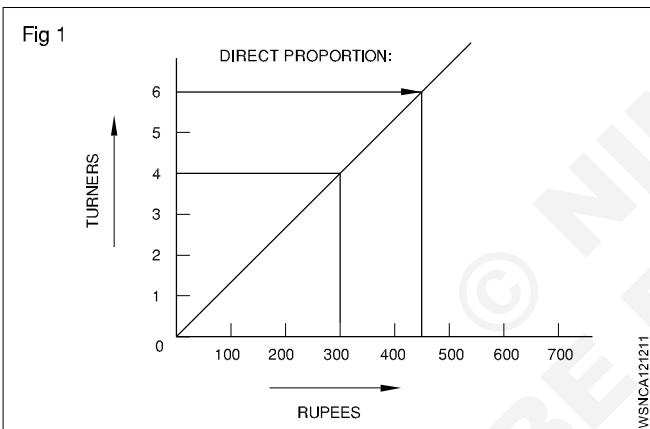
statement

single

multiple.

Direct proportion

The more in one the more in the other - An increase in one denomination produces an increase in the other. (Fig 1)



Examples

- 4 turners earn 300 Rupees. How much will 6 Turners earn?

Statement

4 turners = 300 Rupees

Single

1 Turner = 75 Rupees

Multiple

6 Turners =  $6 \times 75 = 450$  Rupees

- One vehicle consumes 30 litres of petrol per day how much petrol is used by 6 Vehicles operating under similar condition.

One vehicle uses petrol = 30 litres per day.

Then six vehicles will use = 6 Times as much

=  $6 \times 30 = 180$  litres/day.

- 4 vehicles consumes 120 gallons of petrol per day how much petrol will be used by 12 vehicles operating under the same condition.

4 vehicles use 120 gallons per day

1 Vehicle will use  $\frac{120}{4} = 30$  gallons/day

12 vehicles will use  $12 \times 30 = 360$  gallons/day

Both examples are called simple proportion because only two quantities were used and the day is common for both ratios.

- If 2 litres of petrol costs Rs 60. Find the cost of 50 litres.

|                    |                |
|--------------------|----------------|
| Quantity of Petrol | Cost of Petrol |
|--------------------|----------------|

|          |       |
|----------|-------|
| 2 litres | Rs.60 |
|----------|-------|

|           |     |
|-----------|-----|
| 50 litres | $x$ |
|-----------|-----|

1 litre petrol =  $\frac{60}{2} = \text{Rs.}30$

50 litres petrol =  $30 \times 50 = \text{Rs } 1500$

- A 150mm dia gear meshes with 50mm dia gear. If the larger gear has 30 teeth. How many teeth will have the smaller gear have?

|          |              |
|----------|--------------|
| Gear dia | No. of Teeth |
|----------|--------------|

|        |    |
|--------|----|
| 150 mm | 30 |
|--------|----|

|       |     |
|-------|-----|
| 50 mm | $x$ |
|-------|-----|

$x = \frac{50}{150} \times 30 = 10$  teeth.

- A mechanic assembles 7 machines in  $2\frac{1}{2}$  days. How long will it take time to assemble 70 machines at the same rate.

|          |      |
|----------|------|
| Machines | Days |
|----------|------|

|   |                |
|---|----------------|
| 7 | $2\frac{1}{2}$ |
|---|----------------|

|    |     |
|----|-----|
| 70 | $x$ |
|----|-----|

$x = \frac{70 \times 2.5}{7} = 25$  days

Assemble for 70 machines will take 25 days.

- A roll of wire weighs 1.24 kg from this roll a piece of 3.7cm long is cut and it is found to weigh 2.93 gm. What is the length of the wire in the roll?

|                |                |
|----------------|----------------|
| Weight of wire | Length of wire |
|----------------|----------------|

|         |        |
|---------|--------|
| 2.93 gm | 3.7 cm |
|---------|--------|

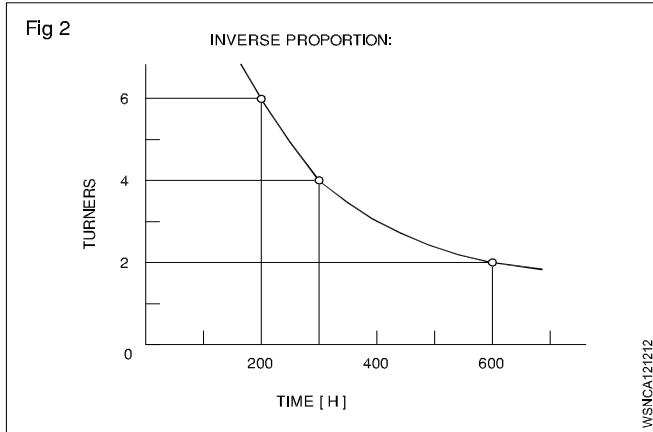
|                   |     |
|-------------------|-----|
| 1.24 kg (1240 gm) | $x$ |
|-------------------|-----|

$$x = \frac{1240}{2.93} \times 3.7 = 1566 \text{ cm}$$

Length of wire = 1566 cm.

### Indirect or inverse proportion

The more in one the lesser other - Increase in one quantity will produce a decrease in the other. (Fig 2)



### Example

- 1 4 turners finish a job in 300 hours. How much time will 6 turners take to do the same job?

Solution procedure in three steps:

Statement 4 turners taken = 300 hours

The time will reduce if 6 turners to do the same job. Therefore this is inverse proportion.

$$\text{Multiple fraction } \frac{4 \text{ Turners}}{6 \text{ Turners}} \times 300 \text{ hours}$$

$$6 \text{ Turners} = 200 \text{ hours}$$

Result - The more the less.

- 2 8 workman take 6 days to complete a job. How many days it will take for 4 workman to complete the same job?

| Workman | Days |
|---------|------|
| 8       | 6    |
| 4       | $x$  |

$$x = \frac{8}{4} \times 6 = 12 \text{ days}$$

4 workers complete the work = 12 days.

- 3 5 men working on a job finished it in 32 days. Find out in how many days 8 men will finish the same job?

| Men | Days |
|-----|------|
| 5   | 32   |
| 8   | $x$  |

$$x = \frac{5 \times 32}{8} = 4 \times 5 = 20 \text{ days}$$

8 men will complete the job = 20 days.

- 4 An engine running at 150 rpm drives a shaft by pulley diameter is 55cm and that of the driven shaft pulley is 33 cm. Find the speed of the shaft?

| Dia of pulley | Rpm of shaft |
|---------------|--------------|
| 55 cm         | 150          |
| 33 cm         | $x$          |

$$x = \frac{55 \times 150}{33} = 250 \text{ rpm.}$$

Speed of the 33cm diameter will run 250 rpm.

- 5 A pulley of 80 cm diameter is rotating at 100 rpm and drives another pulley of 40 cm diameter. Find the rpm of driven pulley. If slip is 2.5% find the rpm?

| Dia of pulley | Rpm of pulley |
|---------------|---------------|
| 80 cm         | 100           |
| 40 cm         | $x$           |

$$40 \text{ cm diameter} = 200 \text{ rpm.}$$

$$\text{Slip is } 2.5\% = 195 \text{ rpm.}$$

### Problems involving both

#### Example

2 turners need 3 days to produce 20 pieces. How long will it take for 6 turners to produce 30 such pieces?

Statement

$$2 \text{ turners, } 20 \text{ pieces} = 3 \text{ days}$$

$$6 \text{ turners, } 30 \text{ pieces} = \text{how many days.}$$

First step (Fig 3)

Statement 2 turners for 20 pieces = 3 days

$$1 \text{ turner for } 20 \text{ pieces} = 3 \times 2 = 6 \text{ days}$$

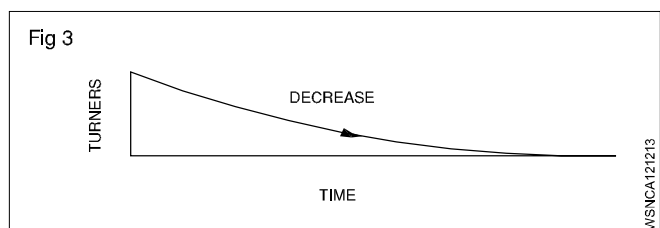
$$\text{Multiple } 6 \text{ turners for } 20 \text{ pieces} = \frac{6}{6} = 1 \text{ day}$$

Statement 6 turners for 20 pieces = 1 day

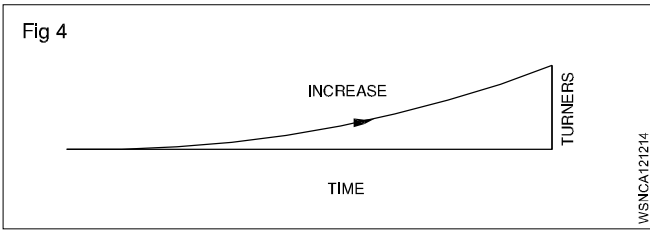
$$\text{Single } 6 \text{ turners for } 1 \text{ piece} = \frac{1}{20} \text{ days}$$

$$\text{Multiple } 6 \text{ turners for } 30 \text{ pieces} = \frac{1}{20} \times 30 = 1.5 \text{ days}$$

### Inverse proportion - More the less.



Second step (Fig 4)



**Direct proportion - More the more.**

**Solve the problem by first writing the statement and proceed to single and then to the multiple according to the type of proportion that is involved.**

### Introduction

Proportional fundamentals, as applicable to motor vehicle calculations are discussed below.

### Simple Proportion

- Proportion

This is an equality between two ratios

### Compound and Inverse proportions

- Compound proportions

### Example

5 Fitter take 21 days to complete overhauling of 6 vehicles how long 7 Fitters will take to over haul 8 vehicles (Assume time of overhauling each vehicle is constant)

In this both direct and indirect proportions are used.

- 1 Fitter will overhauling 1 vehicle in days (shorter time).
- Quantities (No. of days) are taken in last as that is the answer required in this case.

| Fitters | Vehicle | Days |
|---------|---------|------|
| 5       | 6       | 21   |
| 7       | 8       | $x$  |

$$\left(\frac{21 \times 5}{6 \times 7}\right) \times 8 = 20 \text{ days}$$

**Ans:** 7 Fitters will overhaul 8 vehicles in 20 days.

### Inverse proportion

Some times proportions are taken inversely.

### Examples

- If one water pump fills the fuel tank in 12 minutes, two pumps will take half the time taken.

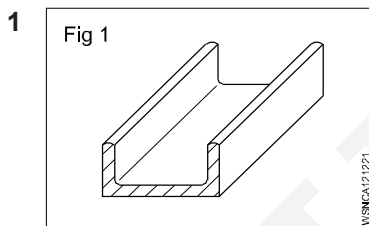
**The time should not be doubled.**

- 2 pumps will take 30 minutes to fill up a tank how long will 6 similar pumps take this to fill this tank.

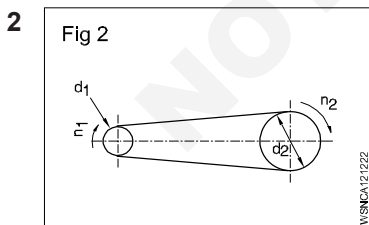
| Pump | Time |
|------|------|
| 2    | 30   |
| 6    | $x$  |

**Ans:** Time taken by 6 pumps =  $\frac{30 \times 2}{6} = 10 \text{ minutes}$

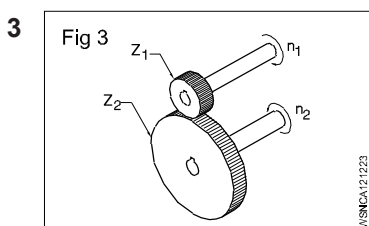
## Assignment



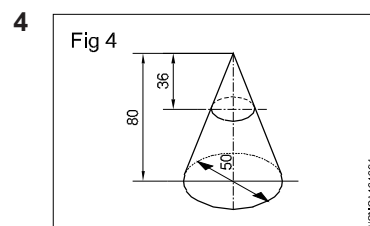
Length = 6.1 metre  
Weight = 32 kgf  
Weight of 1 metre of the same channel = \_\_\_\_\_ kgf



$d_1 = 120 \text{ mm}$   
 $d_2 = 720 \text{ mm}$   
 $n_1 = 1200 \text{ rpm}$   
 $n_2 = \text{_____ rpm}$



$Z_1 = 42 \text{ T}$   
 $n_2 = 96 \text{ rpm}$   
 $n_1 = 224 \text{ rpm}$   
 $Z_2 = \text{_____ T}$



$D = 50 \text{ mm}$   
 $H = 80 \text{ mm}$   
 $h = 36 \text{ mm}$   
 $d = \text{_____ mm}$

- If a mechanic assembles 8 machines in 3 days, how long he will take to assemble 60 machines.
- In an auto shop the grinding wheel makes 1000 rpm and the driven pulley is 200 mm dia. If the driving pulley is 150 mm dia. Find out the rpm of the driving pulley.
- In a gearing of a vehicle the following facts are found. A 180 mm dia of gear meshes with 60mm dia gear. If the bigger gear makes 60 rpm. What will be the rpm of smaller gear.
- A vehicular job is completed by 5 mechanics in 4 days. If only 3 mechanics are available, in how many days the work can be completed.



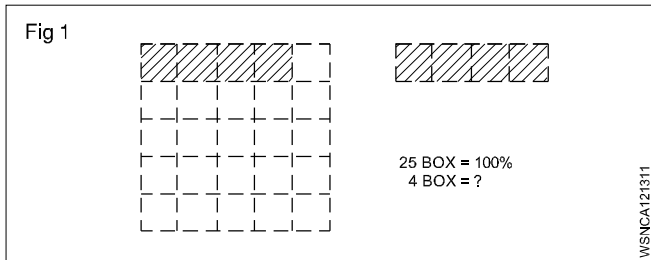
Square root, Ratio and Proportions, Percentage - Percentage

Percentage

Percentage is a kind of fraction whose denominator is always 100. The symbol for percent is %, written after the number. e.g. 16%.

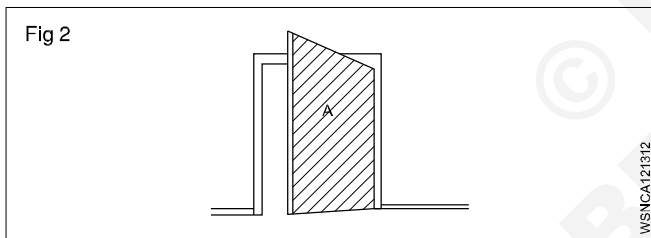
Ex.  $\frac{16}{100} = 0.16$

In decimal form, it is 0.16. Percentage calculation also involves rule of three. The statement (the given data), for unit, and then to multiple which is for calculating the answer. (Fig 1)



Example

The amount of total raw sheet metal to make a door was 3.6 metre<sup>2</sup> and wastage was 0.18 metre<sup>2</sup>. Calculate the % of wastage. (Fig 2)



Solution procedure in three steps.

Statement:

Area of door (A) = 3.6 m<sup>2</sup> = 100 %.

Wastage = 0.18 m<sup>2</sup>

Single:  $\frac{100}{3.6} \times 1 \text{ m}^2$

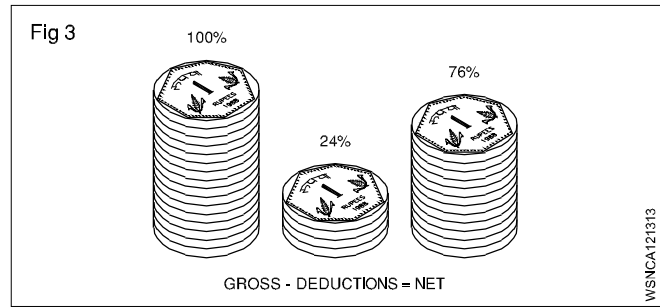
Multiple: for 0.18 m<sup>2</sup> =  $\frac{100}{3.6} \times 0.18$ .

Wastage = 5%.

Analyse the given data and proceed to arrive at the answer through the unit.

Example

A fitter receives a take-home salary of 984.50 rupees. If the deduction amounts to 24%, what is his total salary? (Fig 3)



Total pay 100%

Deduction 24%

Take home salary 76%

If the take home pay is Rs.76, his salary is 100.

For 1% it is  $\frac{1}{76}$

For Rs.984.50, it is  $\frac{1}{76} \times 984.50$ .

For 100% it is  $\frac{984.50}{76} \times 100 = 1295.39$

100% i.e. gross pay = Rs.1295.40.

Example 1

75 litres of oil is taken out from a oil barrel of 200 litres capacity. Find out the percentage taken in this.

Solution

% of oil taken = Oil taken out (litres) / Capacity of Barrel (litres) x 100

$= \frac{75}{200} \times 100 = 37\frac{1}{2}\%$

Example 2

A spare part is sold with 15%. Profit to a customer, to a price of Rs. 15000/-. Find out the following (a) What is the purchase price (b) What is the profit.

Solution: CP = x ,

CP = cost price

SP = sale price

SP=CP+15%of CP

$15000 = x + \frac{15x}{100} = \frac{100x + 15x}{100}$

$x = \frac{1500000}{115} = 13043.47$

Profit = SP-CP = 15000-13043.47 = 1956.53

Purchase price = Rs.13,043/,Profit = Rs. 1957



### Example 3

Out of 80000 cars, which were tested on road, only 16000 cars had no fault. What is the percentage in this acceptance.

$$= \frac{160000}{80000} \times 100 = \frac{100}{5} = 20\%$$

### Example 4

The price of a motor cycle dropped to 92% of original price and now sold at Rs. 18000/- What was the original price.

Solution

Present price of Motor cycle Rs. 18000

This is the value of 92% of original price

$$\text{Original Price} = 18000 \times \frac{100}{92} = \frac{1800000}{92}$$

$$= \text{Rs. } 19565$$

### Example 5

A Motor vehicle uses 100 litres of Petrol per day when travelling at 30 kmph. After top overhauling the consumption falls to 90 litres per day. Calculate percentage of saving.

Solution

Percentage of saving = Decrease in consumption / Original consumption x 100

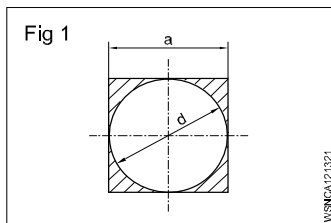
$$= (100 - 90) \frac{\text{litres}}{100} \times 100$$

$$= \frac{10}{100} \times 100$$

= 10% Saving in fuel.

## Assignment

1

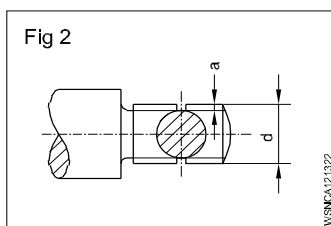


a = 400mm (side of square)

d = 400 mm

Wastage = \_\_\_\_\_ %.

2



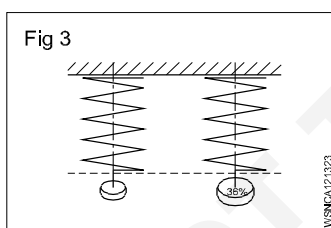
d = 26mm

'a' depth of u/cut = 2.4mm

reduction of area at cross-section

= \_\_\_\_\_ %

3

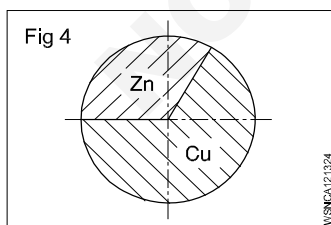


Percentage of increase = 36%

Value of increase = 611.2 N/mm<sup>2</sup>

Original tensile strength = \_\_\_\_\_ N/mm<sup>2</sup>.

4



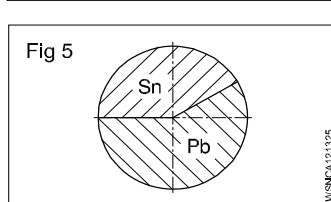
Copper in alloy = 27 kg

Zinc in alloy = 18 kg

% of Copper = \_\_\_\_\_ %

% of Zinc = \_\_\_\_\_ %.

5



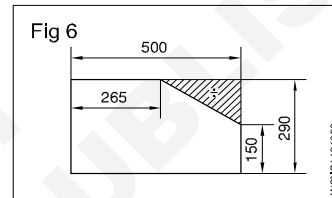
Weight of alloy = 140 Kgf

Weight of Sn 40%

Pb = \_\_\_\_\_ Kgf

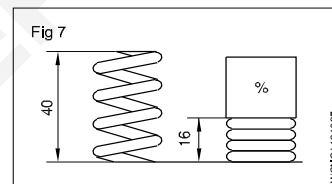
Sn = \_\_\_\_\_ Kgf.

6



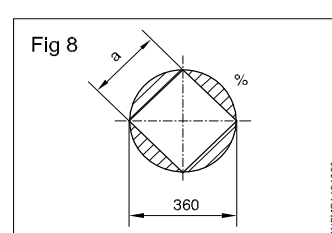
Shaded portion = \_\_\_\_\_ %.

7



Compression length = \_\_\_\_\_ %.

8

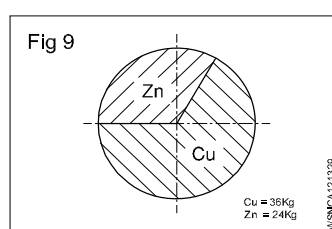


d = 360 mm

a = 0.707 x d

Wastage = \_\_\_\_\_ %.

9



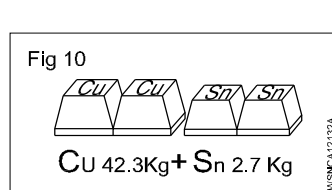
Cu = 36 Kg

Zn = 24 Kg

Cu = \_\_\_\_\_ %

Zn = \_\_\_\_\_ %

10



Cu = 42.3 Kg

Sn = 2.7 Kg

Cu \_\_\_\_\_ %

Sn = \_\_\_\_\_ %.

## Square root, Ratio and Proportions, Percentage - Changing percentage to decimal and fraction

### Conversion of Fraction into Percentage

1 Convert  $\frac{1}{2}$  into percentage.

$$\text{Solution: } \frac{1}{2} \times 100 \\ = 50\%$$

2 Convert  $\frac{1}{11}$  into percentage

$$\text{Solution: } \frac{1}{11} \times 100 = \frac{100}{11} \\ = 9.01\%$$

Convert the following fraction into percentage.

1  $\frac{1}{4}$

2  $\frac{1}{5}$

3  $\frac{2}{3}$

4  $\frac{3}{8}$

### Conversion of Percentage into Fraction

1 Convert 24% into fraction.

$$\text{Solution: } \frac{24}{100} = \frac{6}{25}$$

2 Convert  $33\frac{1}{3}\%$  into fraction.

$$\text{Solution: } \frac{33\frac{1}{3}}{100} = \frac{\frac{100}{3}}{100} = \frac{100}{3} \times \frac{1}{100} \\ = \frac{1}{3}$$

Convert the following percentage into fraction

1 15%

2  $87\frac{1}{2}\%$

3 80%

4 12.5%

### Conversion of Decimal Fraction into Percentage

1 Convert 0.35 into percentage.

$$\text{Solution: } 0.35 \times 100 \\ = 35\%$$

2 Convert 0.375 into percentage.

$$\text{Solution: } 0.375 \times 100 \\ = 37.5\%$$

Convert the following Decimal Fraction into Percentage

1 0.2

2 0.004

3 0.875

4 0.052

### Conversion of Percentage into Decimal fraction

1 Convert 30% into decimal fraction.

$$\text{Solution: } \frac{30}{100} = 0.3$$

2 Convert  $33\frac{1}{3}\%$  into decimal fraction.

$$\text{Solution: } \frac{33\frac{1}{3}}{100} = \frac{\frac{100}{3}}{100} = \frac{100}{3} \times \frac{1}{100} \\ = \frac{1}{3} = 0.333$$

Convert the following percentage into decimal fraction

1 15%

2 7%

3  $12\frac{1}{2}\%$

4 90%

**Material science - Types of metal, types of ferrous and non ferrous metals****Types of metals**

The metals is of two types:

- 1 Ferrous metal                      2 Non-ferrous metal

**1 Ferrous metals :** The metals that contains major part of iron and contain carbon are called ferrous metals such as pig iron, mild steel, nickel etc., they have iron properties such as rusting, magnetisations etc.

**2 Non-ferrous metals :** The metals that do not contains iron or carbon and do not have the property of iron are called non-ferrous metals such as copper, aluminum etc.

**Ferrous and Non ferrous alloys****Alloying metals and ferrous alloys**

An alloy is formed by mixing two or more metals together by melting.

For ferrous metals and alloys, iron is the main constituent metal. Depending on the type and percentage of the alloying metal added, the property of the alloy steel will vary.

**Metals commonly used for making alloy steels****Nickel (Ni)**

This is a hard metal and is resistant to many types of corrosion rust.

It is used in industrial applications like nickel, cadmium batteries, boiler tubes, valves of internal combustion engines, engine spark plugs etc. The melting point of nickel is 1450°C. Nickel can be magnetised. In the manufacture of permanent magnets a special nickel steel alloy is used. Nickel is also used for electroplating. Invar steel contains about 36% nickel. It is tough and corrosion resistant. Precision instruments are made of Invar steel because it has the least coefficient of expansion.

Nickel-steel alloys are available containing nickel from 2% to 50%.

**Chromium (Cr)**

Chromium, when added to steel, improves the corrosion resistance, toughness and hardenability of steel. Chromium steels are available which may contain chromium up to 30%.

Chromium, nickel, tungsten and molybdenum are alloyed for making automobile components and cutting tools.

Chromium is also used for electroplating components. Cylinder liners are chrome-plated inside so as to have wear resistance properties. Stainless steel contains about 13% chromium. Chromium-nickel steel is used for bearings. Chrome-vanadium steel is used for making hand tools like spanners and wrenches.

**Manganese (Mn)**

Addition of manganese to steel increases hardness and strength but decreases the cooling rate.

Manganese steel can be used to harden the outer surface for providing a wear resisting surface with a tough core. Manganese steel containing about 14% manganese is used for making agricultural equipment like ploughs and blades.

**Silicon (Si)**

Addition of silicon for alloying with steel improves resistance to high temperature oxidation.

This also improves elasticity, and resistance against corrosion. Silicon alloyed steels are used in manufacturing springs and certain types of steel, due to its resistance to corrosion. Cast iron contains silicon about 2.5%. It helps in the formation of free graphite which promotes the machinability of cast iron.

**Tungsten (W)**

The melting temperature of tungsten is 3380°C. This can be drawn into thin wires.

Due to this reason it is used to make filaments of electric lamps.

Tungsten is used as an alloying metal for the production of high speed cutting tools. High speed steel is an alloy of 18% tungsten, 4% chromium and 1% vanadium.

Stellite is an alloy of 30% chromium, 20% tungsten, 1 to 4% carbon and the balance cobalt.

**Vanadium (Va)**

This improves the toughness of steel. Vanadium steel is used in the manufacture of gears, tools etc. Vanadium helps in providing a fine grain structure in tool steels.

Chrome-vanadium steel contains 0.5% to 1.5% chromium, 0.15% to 0.3% vanadium, 0.13% to 1.10% carbon.

This alloy has high tensile strength, elastic limit and ductility. It is used in the manufacture of springs, gears, shafts and drop forged components.

Vanadium high speed steel contains 0.70% carbon and about 10% vanadium. This is considered as a superior high speed steel.

**Cobalt (Co)**

The melting point of cobalt is 1495°C. This can retain magnetic properties and wear-resistance at very high temperatures. Cobalt is used in the manufacture of magnets, ball bearings, cutting tools etc. Cobalt high speed steel (sometimes known as super H.S.S.) contains about 5 to 8% cobalt. This has better hardness and wear resistance properties than the 18% tungsten H.S.S.

**Molybdenum (Mo)**

The melting point of molybdenum is 2620°C. This gives high resistance against softening when heated. Molybdenum high speed steel contains 6% of molybdenum, 6% tungsten, 4% chromium and 2% vanadium. This high speed steel is very tough and has good cutting ability.

## **Cadmium (cd)**

The melting point of cadmium is 320°C. This is used for coating steel components.

## **Alloying Metals and Non Ferrous Alloys**

### **Non-ferrous Metals And Alloys**

#### **Copper and its alloys**

Metals without iron are called non-ferrous metals. Eg. Copper, Aluminium, Zinc, Lead and Tin.

#### **Copper**

This is extracted from its ores 'MALACHITE' which contains about 55% copper and 'PYRITES' which contains about 32% copper.

#### **Properties**

Reddish in colour. Copper is easily distinguishable because of its colour.

The structure when fractured is granular, but when forged or rolled it is fibrous.

It is very malleable and ductile and can be made into sheets or wires.

It is a good conductor of electricity. Copper is extensively used as electrical cables and parts of electrical apparatus which conduct electric current.

Copper is a good conductor of heat and also highly resistant to corrosion. For this reason it is used for boiler fire boxes, water heating apparatus, water pipes and vessels in brewery and chemical plants. Also used for making soldering iron.

The melting temperature of copper is 1083°C.

The tensile strength of copper can be increased by hammering or rolling.

#### **Copper Alloys**

##### **Brass**

It is an alloy of copper and zinc. For certain types of brass small quantities of tin or lead are added. The colour of brass depends on the percentage of the alloying elements. The colour is yellow or light yellow, or nearly white. It can be easily machined. Brass is also corrosion-resistant.

Brass is widely used for making motor car radiator core and water taps etc. It is also used in gas welding for hard soldering/brazing. The melting point of brass ranges from 880 to 930°C.

Brasses of different composition are made for various applications.

##### **Bronze**

Bronze is basically an alloy of copper and tin. Sometimes zinc is also added for achieving certain special properties. Its colour ranges from red to yellow. The melting point of bronze is about 1005°C. It is harder than brass. It can be easily machined with sharp tools. The chip produced is granular. Special bronze alloys are used as brazing rods.

Bronze of different compositions are available for various applications.

#### **Lead and its alloys**

Lead is a very commonly used non-ferrous metal and has a variety of industrial applications.

Lead is produced from its ore 'GALENA'. Lead is a heavy metal that is silvery in colour when molten. It is soft and malleable and has good resistance to corrosion. It is a good insulator against nuclear radiation. Lead is resistant to many acids like sulphuric acid and hydrochloric acid.

It is used in car batteries, in the preparation of solders etc. It is also used in the preparation of paints.

#### **Lead Alloys**

Babbitt metal

Babbitt metal is an alloy of lead, tin, copper and antimony. It is a soft, anti-friction alloy, often used as bearings.

An alloy of lead and tin is used as 'soft solder'.

#### **Zinc and its alloys**

Zinc is a commonly used metal for coating on steel to prevent corrosion. Examples are steel buckets, galvanized roofing sheets, etc.

Zinc is obtained from the ore-calamine or blende.

Its melting point is 420° C.

It is brittle and softens on heating; it is also corrosion-resistant. Due to this reason it is used for battery containers and is coated on roofing sheets etc.

Galvanized iron sheets are coated with zinc.

#### **Tin and tin alloys**

##### **Tin**

Tin is produced from cassiterite or tinstone. It is silvery white in appearance, and the melting point is 231° C. It is soft and highly corrosion-resistant.

It is mainly used as a coating on steel sheets for the production of food containers. It is also used with other metals, to form alloys.

**Example:** Tin with copper to form bronze. Tin with lead to form solder. Tin with copper, lead and antimony to form Babbitt metal.

#### **Aluminium**

Aluminium is a non-ferrous metal which is extracted from 'BAUXITE'. Aluminium is white or whitish grey in colour. It has a melting point of 660° C. Aluminium has high electrical and thermal conductivity. It is soft and ductile, and has low tensile strength. Aluminium is very widely used in aircraft industry and fabrication work because of its lightness. Its application in the electrical industry is also on the increase. It is also very much in use in household heating appliances.

Material science - Physical and mechanical properties of metals

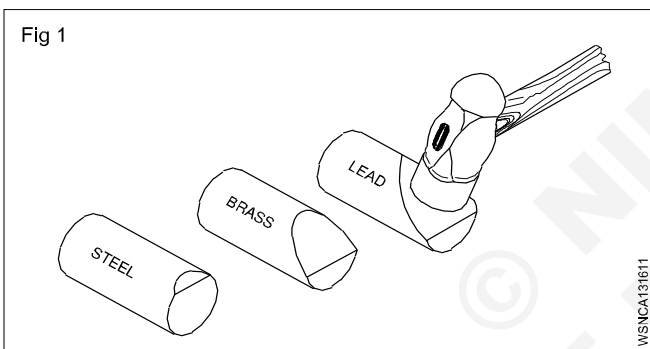
**Metal:**

Metal is a mineral used in all types of engineering works such as machineries, bridges, aero planes etc., so we must have basic knowledge about the metals.

Understanding the physical and mechanical properties of metals has become increasingly important for a machinist since he has to make various components to meet the designed service requirements against factors, such as the raise of temperature, tensile, compressive and impact loads etc. A knowledge of different properties of materials will help him to do his job successfully. If proper material/metal is not used it may cause fracture or other forms of failures, and endanger the life of the component when it is put into function.

Fig 1 shows the way in which the metals get deformed when acted upon by the same load.

Note the difference in the amount of deformation.



**Physical properties of metals**

- Colour
- Weight/specific gravity
- Structure
- Conductivity
- Magnetic property
- Fusibility

**Colour**

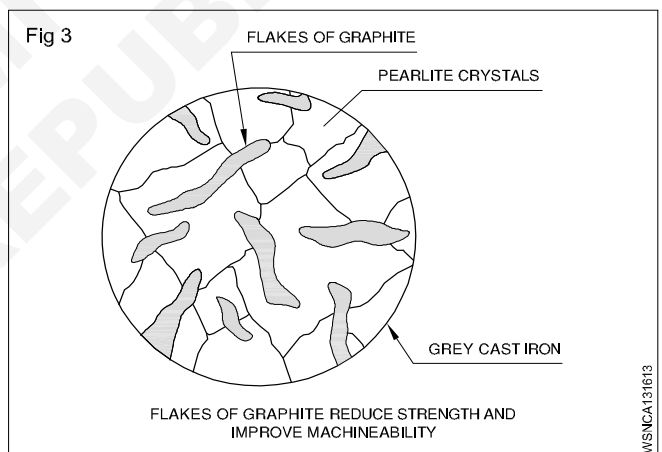
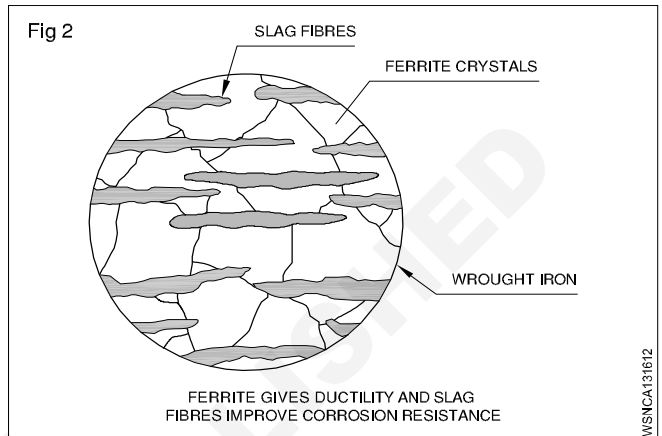
Different metals have different colours. For example, copper is distinctive red colour. Mild steel is blue/black sheen.

**Weight**

Metals may be distinguished, based on their weights for given volume. Metals like aluminium lighter weight (Specific gravity 2.7) and metals like lead have a higher weight. (Specific gravity 11.34)

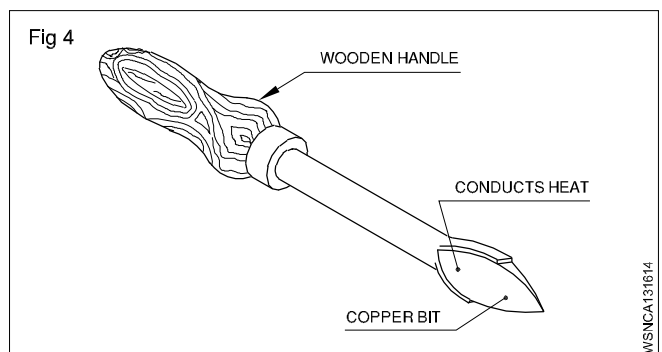
**Structure (Figs 2&3)**

Generally metals can also be differentiated by their internal structures while seeing the cross-section of the bar through a microscope. Metals like wrought iron and aluminium have a fibrous structure and metals like cast Iron and bronze have a granular structure.



**Conductivity (Figs 4&5)**

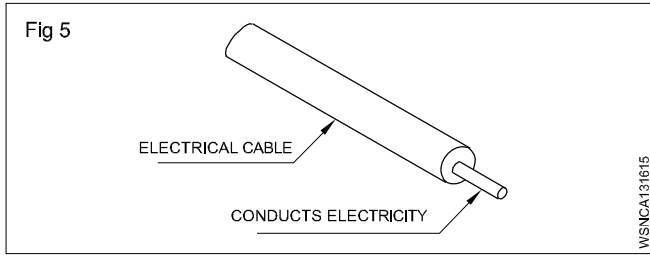
Thermal conductivity and electrical conductivity are the measures of ability of a material to conduct heat and electricity. Conductivity will vary from metal to metal. Copper and aluminium are good conductors of heat and electricity.



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### Magnetic property

A metal is said to possess a magnetic property if it is attracted by a magnet.

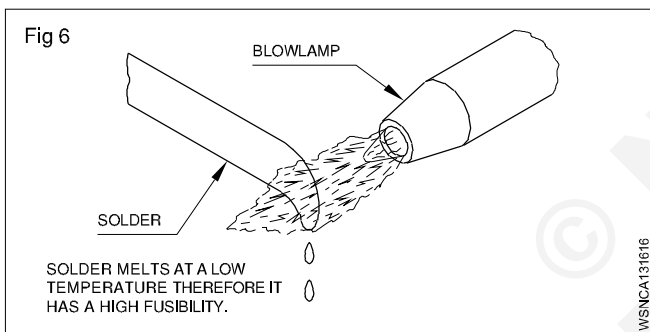
Almost all ferrous metals, except some types of stainless steel, can be attracted by a magnet, and all non-ferrous metals and their alloys are not attracted by a magnet.

### Fusibility (Fig 6)

It is the property possessed by a metal by virtue of which it melts when heat is applied. Many materials are subject to transformation in the shape (i.e) from solid to liquid at different temperatures. Lead has a low melting temperature while steel melts at a high temperature.

Tin melts at 232°C.

Tungsten melts at 3370°C.

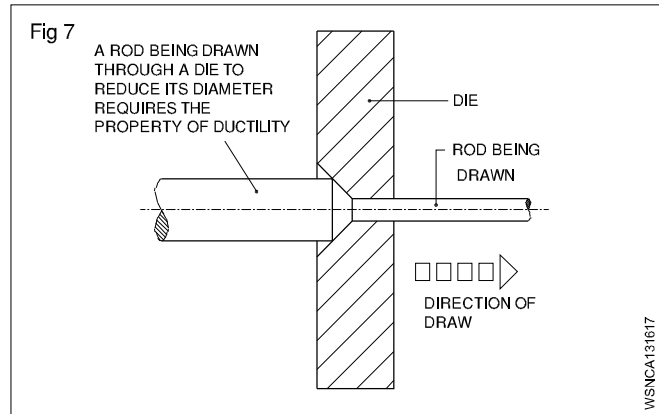


### Mechanical properties

- Ductility
- Malleability
- Hardness
- Brittleness
- Toughness
- Tenacity
- Elasticity

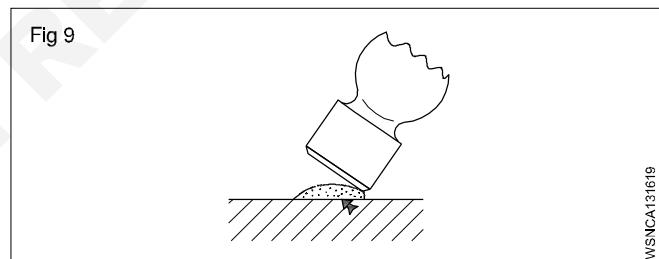
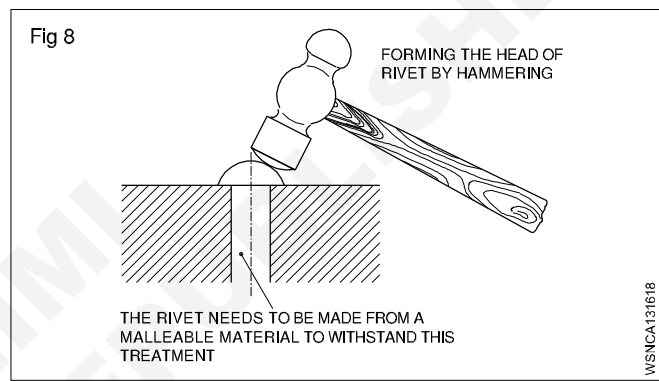
### Ductility (Fig 7)

A metal is said to be ductile when it can be drawn out into wires under tension without rupture. Wire drawing depends upon the ductility of a metal. A ductile metal must be both strong and plastic. Copper and aluminium are good examples of ductile metals.



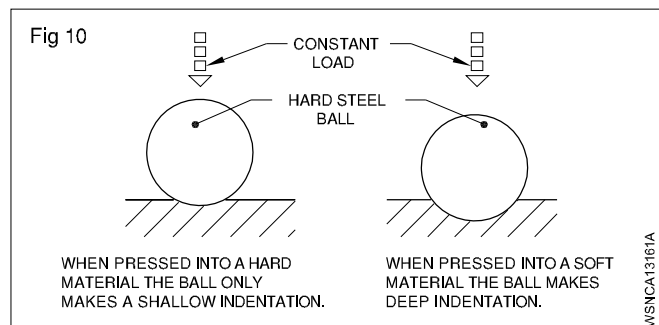
### Malleability (Figs 8 and 9)

Malleability is the property of a metal by which it can be extended in any direction by hammering, rolling etc. without causing rupture. Lead is an example of a malleable metal.



### Hardness (Fig 10)

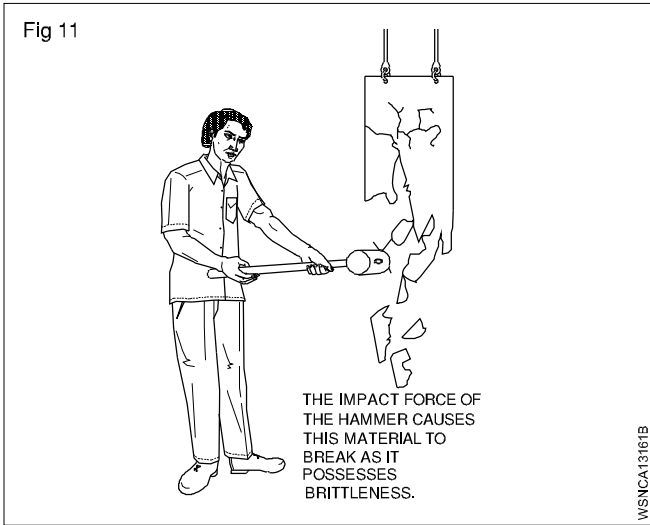
Hardness is a measure of a metal's ability to withstand scratching, wear and abrasion, indentation by harder bodies. The hardness of a metal is tested by marking by a file etc.





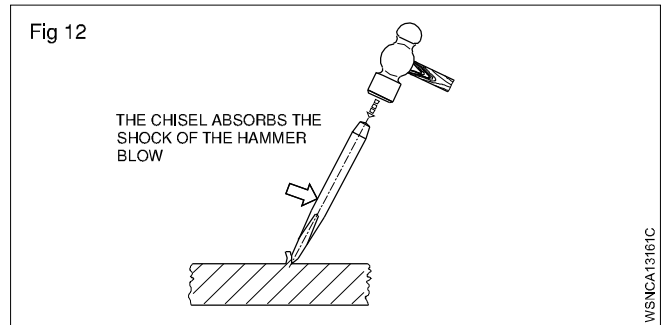
### Brittleness (Fig 11)

Brittleness is that property of a metal which permits no permanent distortion before breaking. Cast iron is an example of a brittle metal which will break rather than bend under shock or impact.



### Toughness (Fig 12)

Toughness is the property of a metal to withstand shock or impact. Toughness is the property opposite to brittleness. Wrought iron is an example of a tough metal.



### Tenacity

The tenacity of a metal is its ability to resist the effect of tensile forces without rupturing. Mild steel, Wrought Iron and copper are some examples of tenacious metals.

### Elasticity

Elasticity of a metal is its power of returning to its original shape after the applied force is released. Properly heat-treated spring is a good example for elasticity.

**Material science - Properties and uses of timber****Properties and uses of timber****General properties**

Timber should have the following properties

- Straight fibres.
- Silky lustre when planed.
- Uniform colour.
- Regular annual rings.
- Heaviness.
- Firm adhesion of fiber and compact modulary rays.
- Sweet smell.
- It should be free from loose or dead knots and shakes.
- The surface should not clog the teeth of the saw on cutting but should remain bright.

**Classification**

- Timbers are classified as
  - a Softwood
  - b hardwood

**Softwood timber**

- Usually all trees with needle leaves of softwood and those with broad leaves are of hard-wood.
- The wood contains resins and turpentine.
- The wood has a fragrant smell.
- Fibres are straight.
- Texture is soft and regular.
- Tough for resisting tensile stresses.
- Weak across the fibres.
- Annual rings are distinct, having one side soft, porous and light coloured. The other side is dense and dark.

- The general colour of the wood is pale tinted or light such as pine spruce, fir, ash, kail, deodar etc.

**Properties of hardwood**

- The wood generally contains a large percentage of acid.
- It is brightly coloured.
- Annual rings are not distinct.
- It is difficult and hard to work with.
- It resists shearing stress.
- Fibre are overlapped.
- The general colour is dark brown such as oak, walnut, teak, mahagony, sishim, babul, sal etc.

**Uses****Soft timber**

- Because of its cheapness it is used for low grade furniture, doors and windows for cheap type of houses.
- Used as fuel.
- Some timbers are used for baskets and mat making.
- The bark is used as garment in some places.

**Hard timber**

- Used for high quality furniture such as chairs, tables, sofas, dewans, beds, etc.
- Used for door, window frames for high quality houses as they can take good polish and painting finish.
- Used for manufacturing katha.

**Wood as an electrical insulator**

Wood is impregnated with oil or other substance, for use as insulator.

Example

It is used in electrical machine windings, as slot wedges.



**Mass, Weight, Volume and Density - Mass, volume, density, weight and specific gravity**

**Mass**

Mass of a body is the quantity of matter contained in a body. The unit of mass in F.P.S system is pound (lb), in C.G.S. system gram (gr) and in M.K.S and S.I systems kilogram (kg). 1ton which is 1000kg is also used sometimes. The conversion factor is 1000. Three decimal places are shifted during conversion. E.g. 1 ton = 1000 kg 1g = 1000mg.

m - mass of a body

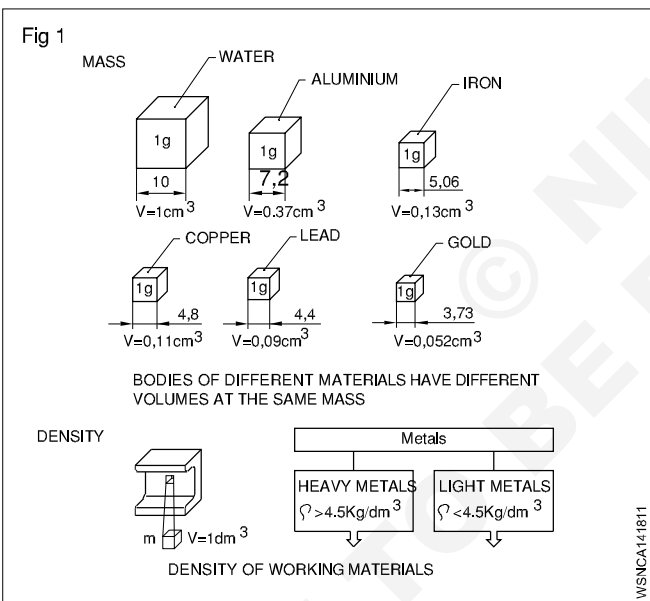
g - acceleration due to gravity in metre/sec<sup>2</sup> = 9.81 m/sec<sup>2</sup>

V - volume of the body

ρ - density (pronounced as 'rho')

W or FG - weight or weight force

**Mass (Fig 1)**



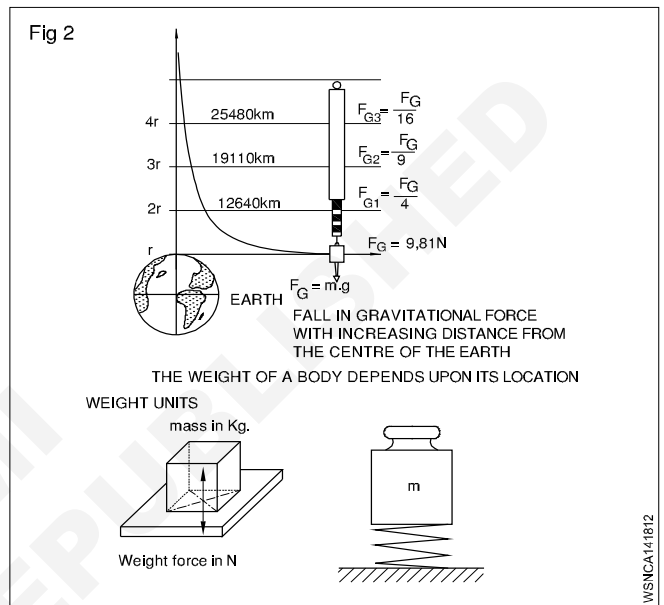
**Density**

Density is the mass of a body per unit volume. Hence its unit will be gr/cm<sup>3</sup> or kg/dm<sup>3</sup> or ton/m<sup>3</sup>.

$$\text{Density} = \frac{\text{mass}}{\text{volume}} = \frac{m}{v} = \rho$$

**Weight (Fig 2)**

Weight is the force with which a body is attracted by the earth towards its centre. It is the product of the mass of the body and the acceleration due to gravity. The weight of a body depends upon its location.



weight = W or FG = mass x gravitational force = m x g

| System        | Absolute unit                          | Derived unit | Conversion   |
|---------------|--|--------------|--|
| F.P.S. system | 1 poundal                              | 1 Lb wt      | 32.2 poundals (1 lb x 1 ft/sec <sup>2</sup> = 1 pound) |
| C.G.S. system | 1 dyne<br>1 gr x 1 cm/sec <sup>2</sup> | 1 Gr. wt     | 981 dynes  |
| M.K.S.        | Newton                                 | 1 kg. wt     | 1 Newton =   |
| S.I. system   | Newton                                 | Newton       | 1 kg x 1 m/sec <sup>2</sup>                            |

**1 kg. wt = 9.81 Newton (approximately 10N)**      **1 Newton = 10<sup>5</sup> dynes.**

## Difference between mass and weight

| S. No | Mass  | Weight   |
|-------|---|--|
| 1     | Mass is the quantity of matter in a body (ie) measurement of matter in a body | Weight is measure of amount of force acting on mass due to acceleration due to gravity |
| 2     | It does not depend on the position or space                                   | It depends on the position, location and space   |
| 3     | Mass of an object will not be zero  | Weight of an object will be zero if gravity is absent                                  |
| 4     | It is measured using by physical balance                                      | It is measured using by spring balance   |
| 5     | It is a scalar quantity   | It is a vector quantity  |
| 6     | When immersed in water mass does not change                                   | When immersed in water weight will change  |
| 7     | The unit is in grams and kilogram   | The unit is in kilogram weight, a unit of force  |

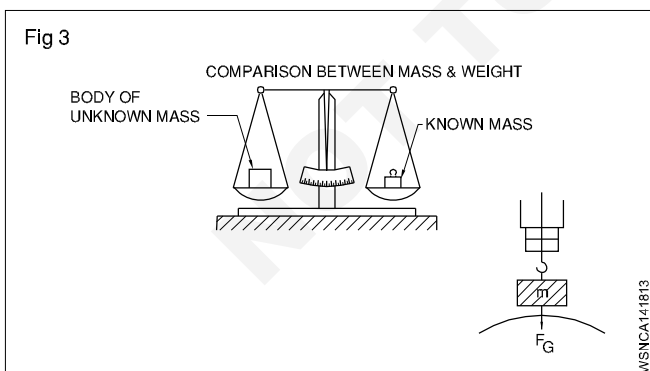
**Mass and weight are different quantities.**

**Mass of a body is equal to volume x density.**

**Weight force is equal to mass x acceleration due to gravity.**

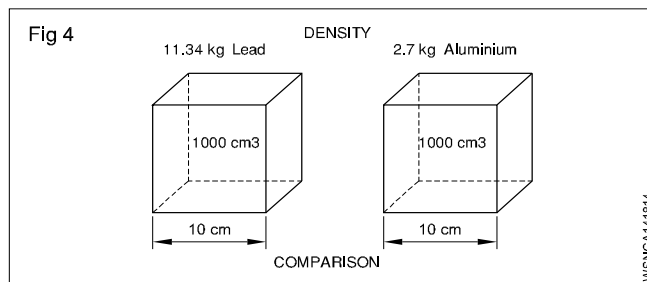
## Weight , Density and Specific gravity

It is now seen that the mass of a substance is measured by its weight only without any reference to volume. But if equal weights of lead & aluminium, are compared the volume of lead is much smaller than volume of aluminium. So we can now say that lead is more dense than aluminium, .i.e In other words the density of lead is greater than aluminium. (Fig 3 & 4)



The relation of mass and volume is called density.

The density expresses the mass of volume E.g. 1 dm<sup>3</sup> of water has the mass of 1kg - thus the density of 1kg/dm<sup>3</sup> (Fig 2)



## Unit

The density is measured as below

MKS/SI= Kg/m<sup>3</sup>, CGS = 1 gm/cm<sup>3</sup> FPS = lbs/c ft

| Solids |           | gm/cc        | Liquids    | gm/cc |
|--------|-----------|--------------|------------|-------|
| 1      | Aluminum  | 2.7          | Water      | 1.00  |
| 2      | Lead      | 11.34        | Petrol     | 0.71  |
| 3      | Cast iron | 6.8 to 7.8   | Oxygen     | 1.43  |
| 4      | Steel     | 7.75 to 8.05 | Diesel Oil | 0.83  |

The specific gravity of a substance is also called its relative density.

Formula

Specific gravity (or) Relative density =  $\frac{\text{Density of the substance}}{\text{Density of the water at } 4^{\circ}\text{C}}$

$$= \frac{\text{Mass of any volume of a substance}}{\text{Mass of an equal volume of water at } 4^{\circ}\text{C}}$$

## Comparison Between Density And Specific Gravity (Relative Density)

| Density   | Relative density or Specific gravity  |
|---|---|
| Mass per unit volume of a substance is called its density<br>Its unit is gm per cu cm; lbs per cu.ft and kg/cubic meter | The density of substance to density of water at 4°C is its relative density<br>It has no unit of measurement simply expressed in a number |
| Density = $\frac{\text{Mass}}{\text{Volume}}$   | Relative density = $\frac{\text{Density of the substance}}{\text{Density of water at } 4^{\circ}\text{C}}$                                |

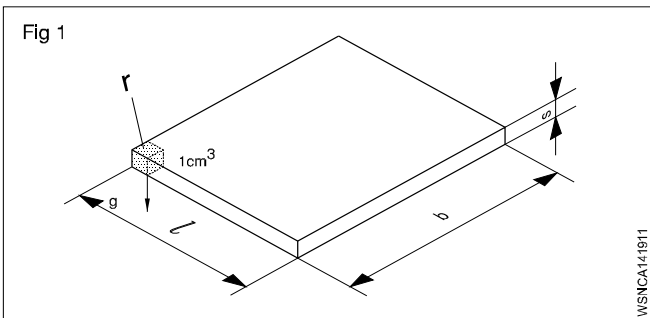
| Solids |           | Sp.gy      | Liquids      | Sp.gy       |
|--------|-----------|------------|--------------|-------------|
| 1      | Aluminium | 2.72       | Petrol       | 0.71        |
| 2      | Lead      | 11.34      | Battery acid | 1.2 to 1.23 |
| 3      | Cast iron | 6.8 to 7.8 | Water        | 1.00        |
| 4      | Steel     | 7.82       | Diesel Oil   | 0.83        |

From the above table, we can calculate the weight of any given volume of a substance (say Diesel oil) in any units provided we know the specific gravity of the substance. Also vice-versa for volume of density is known.

**Mass, Weight, Volume and Density - Related problems for mass, volume, density, weight and specific gravity**

- 1 Calculate the mass in kg of a rectangular steel plate of dimensions 220 x 330 x 15 mm (Fig 1) (density of steel = 7.82 gm/cm<sup>3</sup>)

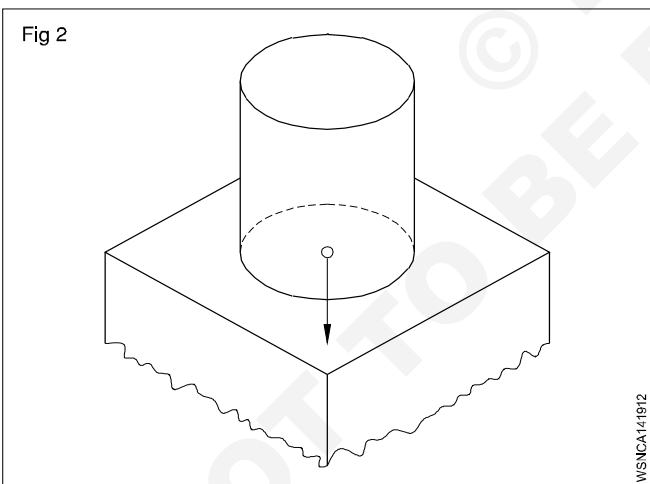
Mass = Volume x density  
 = 22 x 33 x 1.5cm x 7.82 gm/cm<sup>3</sup>  
 = 1089 cm<sup>3</sup> x 7.82 gm/cm<sup>3</sup>  
 mass = 8.516 kg



- 2 What is the weight in Newton if a storage container holds 250 litres of water and amount of water exert on the surface which it is standing? (Fig 2)

( 1 litre of water = 1 kg of water )

Density of water 1 gm/cm<sup>3</sup> or 1 kg/dm<sup>3</sup>



**Acceleration due to gravity is taken as 10 metre/sec<sup>2</sup> (approximation).**

Capacity = 250 litres = 250 dm<sup>3</sup> in volume.  
 Mass of water = volume x density of water  
 = 250 dm<sup>3</sup> x 1 kg/dm<sup>3</sup> = 250 kg  
 Weight extended = mass x acceleration due to gravity  
 = 250 kg x 10 metre/sec<sup>2</sup>  
 = 2500 kg.metre/sec<sup>2</sup> = 2500 N (∵ 1 kg.m/sec<sup>2</sup>=1N)

- 3 A force of 15 dynes acting on a mass of 'm' produces an acceleration of 2.5 cm/sec<sup>2</sup>. Find the mass.

1 Gr. wt. = 981 dynes

∴ 15 dynes =  $\frac{15}{981}$  Gr.wt

Force = m x acceleration produced by the force

∴ Gr.wt = mass x 2.5 cm/sec<sup>2</sup>

∴ gr.cm/sec<sup>2</sup> = mass x 2.5 cm/sec<sup>2</sup>

∴ mass =  $\frac{15}{981 \times 2.5}$  grams =  $\frac{\text{gm.cm/sec}^2}{\text{cm/sec}^2}$

mass = 0.00612 gram

- 4 A force of 2 N acts on a mass of 10 kg. Find the acceleration produced by the force on the mass.

Force = 2 N (∵ 1 N = 1kg.m/sec<sup>2</sup>)

Force = mass x acceleration

∴ 2 kg.metre/sec<sup>2</sup> = 10 kg x acceleration produced

∴ 2 x 1 kg.metre/sec<sup>2</sup> = 10 kg x acceleration produced

∴ acceleration produced =  $\frac{2}{10}$  metre/sec<sup>2</sup>

= 0.2 metre/sec<sup>2</sup>

- 5 Calculate the weight of a body having a mass of 1 kg if the acceleration due to gravity is 9.81 metre/sec<sup>2</sup>

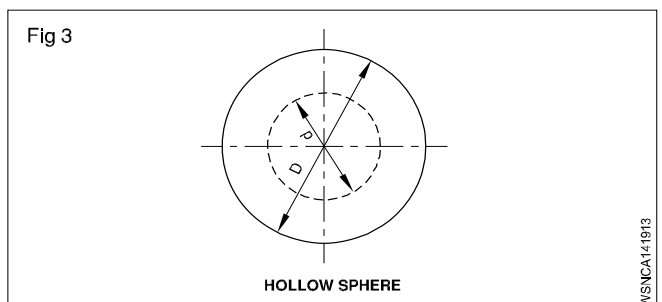
Weight force = mass x acceleration due to gravity  
 = 1 kg x 9.81 metre/sec<sup>2</sup>

(1 kg.metre/sec<sup>2</sup> = 1 N)

∴ 9.81 kg metre/sec<sup>2</sup> = 9.81 N

**In the examples solved the value of 'g' is taken as 10 metre/sec<sup>2</sup>, unless specifically mentioned otherwise.**

- The outside and inside diametres of a hollow sphere are 150 & 70mm respectively. Calculate its mass if the density of material is 7.5 gm/cm<sup>3</sup>. (Fig 3)



Mass = Volume x Density

= Volume x 7.5 gm/cm<sup>3</sup>

D= 150 mm = 15cm R= 7.5 cm

$$d = 70\text{mm} = 7 \text{ cm} \quad r = 3.5 \text{ cm}$$

$$\text{Volume} = \frac{4}{3}\pi(R^3 - r^3)\text{unit}^3$$

$$= \frac{4}{3}\pi(7.5^3 - 3.5^3)$$

$$= 1587.5 \text{ cm}^3$$

$$\text{Mass} = 1587.5 \text{ cm}^3 \times 7.5 \text{ gm/cm}^3$$

$$= 11906.6 \text{ gm} = 11.9\text{kg} \quad \text{say } 12\text{kg}$$

6 A car has a mass of 800 kg. Find out its weight force (Take 9.81 m/sec<sup>2</sup>)

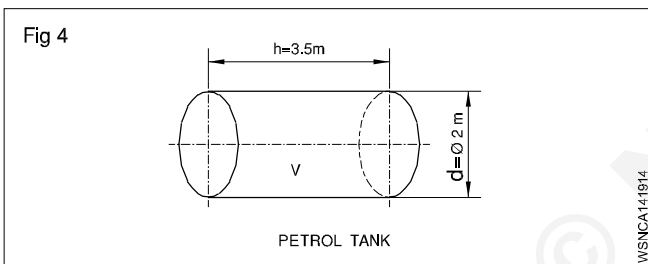
$$(\because 1\text{n} = 1\text{kg}\cdot\text{m}/\text{sec}^2)$$

The Wt. force of a car = Mass of car x gravitational acceleration

$$= 800 \times 9.81 \text{ N}$$

$$= 7848 \text{ Newtons}$$

7 A cylindrical tank 2m dia x 3.5 m length is filled with petrol. Find the weight of petrol in Tonnes, Assume density of petrol 720 Kg/m<sup>3</sup>. (Fig 4)



Volume of Tank

$$v = \pi r^2 l \text{ (or)} \frac{\pi d^2}{4} \times l \text{ unit}^3 = \frac{\pi \times 2^2}{4} \times 3.5 \text{ m}^3$$

$$3.14 \times 3.5 \text{ m}^3 = 10.99 \text{ m}^3$$

$$\text{Since } 1 \text{ m}^3 = 1000 \text{ litres}$$

$$\text{Volume of Tank} = 10.99 \times 1000 \text{ litres}$$

$$\text{Density of petrol} = 720 \text{ Kg/m}^3.$$

$$\text{Weight of Petrol in Kg} = 10.99 \times 1000 \text{ litres} \times 720 \text{ Kg}$$

$$= 720 \times 10990 \text{ Kg}$$

Weight of Petrol in Tonnes

$$\text{(Metric Units)} = \frac{720 \times 10990}{1000}$$

$$\text{Weight of Petrol} = 7912.8 \text{ Tonnes}$$

8 If the battery acid specific gravity is 1.3, and this is being filled up into a cylindrical tank. Find out its density.

$$\text{(Density of water} = 1000 \text{ gm/cm}^3)$$

Specific gravity or Relative density

$$= \frac{\text{Density of the substance}}{\text{Density of water at } 4^\circ\text{c}}$$

Now, density of battery acid

$$= \text{Specific gravity} \times \text{Density of water}$$

$$= 1.3 \times 1000 \text{ gm/cm}^3$$

$$= 1300 \text{ gm/cm}^3$$

### Determination of specific gravity of a substance

The specific gravity of a substance may be determined by

1 Archimedes Principle

2 Hydrometer

### Archimedes Principle

Archimedes principle states that when a body is fully or partially immersed in a liquid, the amount of liquid displaced by the body is equal to the loss of weight of the body in the liquid.

Weight of a body in a liquid = total weight of the body

- weight of the liquid displaced by the body

This quantity if it is zero then the body will float. It is negative the body will rise up till the weight of liquid displaced by the immersed portion of the body is equal and equal to the weight of the body. If it is positive the body will sink. Specific gravity of solids soluble in water

$$= \frac{\text{weight of solid in air}}{\text{loss of weight of solid in water}}$$

specific gravity of solids soluble in water

$$= \frac{\text{weight of solid in air} \times \text{specific gravity of the liquid}}{\text{loss of weight of solid in which the solid is in solution}}$$

specific gravity of a liquid

$$= \frac{\text{loss weight of a solid in water}}{\text{loss of weight of the same solid in liquid}}$$

**The solid chosen should be such that it is insoluble in both water and the liquid whose specific gravity is to be determined.**

### Example

1 An iron piece weighs 160 kgf in air and 133 kgf when it is fully immersed in water. Determine the volume and specific gravity of the iron piece.

$$\text{Weight of the solid in air} = 160 \text{ kgf}$$

$$\text{Weight of the solid in water} = 133 \text{ kgf}$$

$$\therefore \text{Loss of weight in water} = 27 \text{ kgf}$$

By Archimedes principle the loss of weight of a solid in water = volume of water displaced.

$$\therefore \text{Volume of water displaced} = 27 \text{ cm}^3$$

$$\therefore \text{Volume of the solid} = 27 \text{ cm}^3$$

$$\text{Density of the iron piece} = \frac{\text{mass of iron}}{\text{volume of the piece}}$$

$$= \frac{160}{160 - 133} = \frac{160}{27} = 5.93$$

$$\text{Specific gravity} = \frac{\text{density of iron}}{\text{density of water}} = \frac{5.93}{1} = 5.93$$

Specific gravity of iron piece = 5.93

- 2 A metal piece weighs 6.5 kgf in air and 3.5 kgf in water. Find its weight when it is fully immersed in a liquid whose specific gravity is 0.8 and also the S.G of the metal.

Weight of metal piece in air = 6.5 kgf

Weight of metal piece in water = 3.5 kgf

$$\therefore \text{Loss of weight in water} = 3.00 \text{ kgf} (6.5 - 3.5)$$

$\therefore$  Specific gravity of metal

$$= \frac{\text{Weight of substance in air}}{\text{Loss of weight in water}} = \frac{6.5 \text{ kgf}}{3 \text{ kgf}} = 2.166$$

By applying the principle of Archimedes the above results are derived.

The weight of the metal piece in the liquid = W

$$\therefore \text{loss of weight of the metal in the liquid} = 6.5 \text{ kgf} - W$$

$$\text{Specific gravity of the liquid} = \frac{\text{loss of weight in liquid}}{\text{loss of weight of water}}$$

$$\therefore 0.8 = \frac{6.5 \text{ kgf} - W}{3 \text{ kgf}}$$

$$\therefore W = 6.5 \text{ kgf} - 3 \text{ kgf} \times 0.8 = 4.1 \text{ kgf}$$

$$\therefore \text{loss of weight of the metal in the liquid} = 4.1 \text{ kgf.}$$

By using a hydrometer, the specific gravity of a liquid is determined. The most common type of hydrometer is the Nicholson's hydrometer which is a variable weight but constant immersion type.

Specific gravity of a liquid

wt. of hydrometer+ wt. required to sink the hydrometer in the liquid to a fixed mark

-----  
wt. of hydrometer+ wt. required to sink the hydrometer in water up to the same mark

- 3 A piece of wood having 80cc volume, when floated on water remains 60cc volume above water. Find the weight of the water displaced.

Water displaced = Immersed volume

Immersed volume = volume of wood - volume of wood above water level

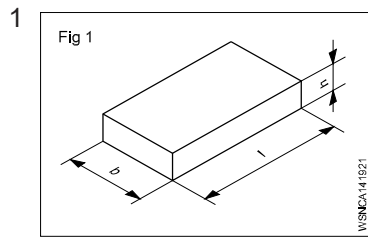
$$= 80 - 60 = 20 \text{ c.c}$$

Weight of water displaced = volume x density of water

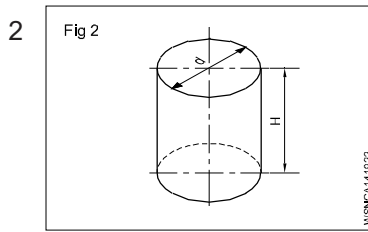
$$= 20 \times 1 = 20 \text{ grams}$$

| Sl.No | Substance         | Density gm/cc |
|-------|-------------------|---------------|
| 1     | Aluminium         | 2.7           |
| 2     | Cast Iron         | 6.8 - 7.8     |
| 3     | Copper            | 8.92          |
| 4     | Gold              | 19.32         |
| 5     | Iron              | 7.86          |
| 6     | Lead              | 11.34         |
| 7     | Nickel            | 8.912         |
| 8     | Silver            | 10.5          |
| 9     | Steel             | 7.75 - 8.05   |
| 10    | Tin               | 7.31          |
| 11    | Zinc              | 7.14          |
| 12    | Diamond           | 3.51          |
| 13    | Bismuth           | 9.78          |
| 14    | Brass             | 8.47          |
| 15    | Phosphrous Bronze | 8.7 - 8.9     |
| 16    | Ice               | 0.93          |
| 17    | Air               | 0.0013        |
| 18    | Mercury           | 13.56         |
| 19    | Petrol            | 0.71          |
| 20    | Diesel            | 0.83          |
| 21    | Kerosene          | 0.78 - 0.81   |
| 22    | Water             | 1.0           |

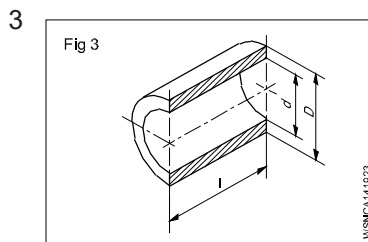
# Assignment



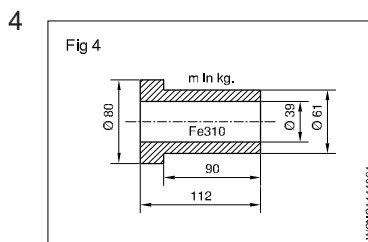
$l = 1800 \text{ mm}$   
 $b = 65 \text{ mm}$   
 $h = 12 \text{ mm}$   
 $\rho = 7.85 \text{ g/cm}^3$   
 $m = \underline{\hspace{2cm}} \text{ kg}$



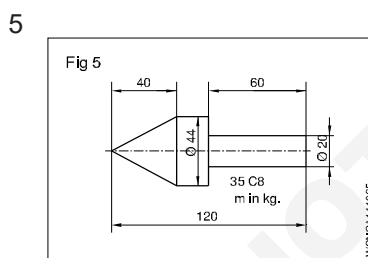
Capacity = 36 litres  
 $d = 32 \text{ cm}$   
 $H = \underline{\hspace{2cm}} \text{ cm}$



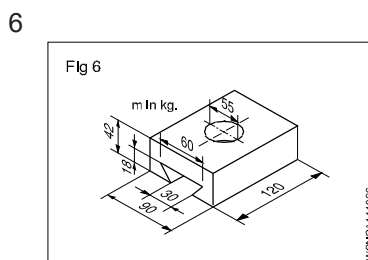
$D = 74 \text{ mm}$   
 $d = 68 \text{ mm}$   
 $l = 115 \text{ mm}$   
 $\rho = 8.6 \text{ g/cm}^3$   
 $m = \underline{\hspace{2cm}} \text{ gms}$



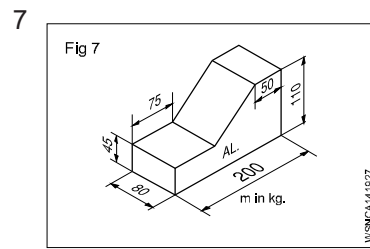
$D_1 = 80 \text{ mm}$   
 $D_2 = 61 \text{ mm}$   
 $d = 39 \text{ mm}$   
 $L = 112 \text{ mm}$   
 $l = 90 \text{ mm}$   
 $\rho = 7.85 \text{ g/cm}^3$   
 $m = \underline{\hspace{2cm}} \text{ kg}$



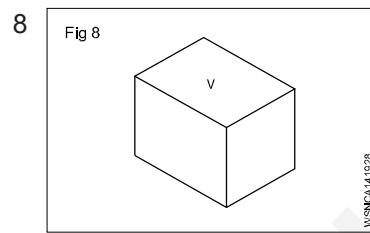
$D = 44 \text{ mm}$   
 $d = 20 \text{ mm}$   
 $L = 120 \text{ mm}$   
 $l_1 = 60 \text{ mm}$   
 $l_2 = 40 \text{ mm}$   
 $\rho = 7.85 \text{ g/cm}^3$   
 $m = \underline{\hspace{2cm}} \text{ kg}$



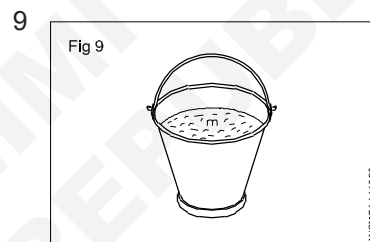
$L = 120 \text{ mm}$   
 $B = 90 \text{ mm}$   
 $b_1 = 60 \text{ mm}$   
 $b_2 = 30 \text{ mm}$   
 $d = 55 \text{ mm}$   
 $H = 42 \text{ mm}$   
 $h = 18 \text{ mm}$   
 $\rho = 7.85 \text{ g/cm}^3$   
 $m = \underline{\hspace{2cm}} \text{ kg}$



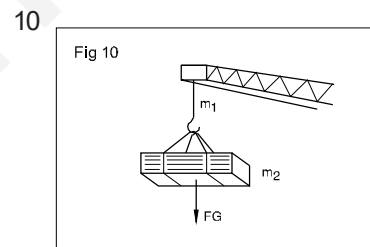
$L = 200 \text{ mm}$   
 $l_1 = 75 \text{ mm}$   
 $l_2 = 50 \text{ mm}$   
 $B = 80 \text{ mm}$   
 $H = 110 \text{ mm}$   
 $h = 45 \text{ mm}$   
 $\rho = 2.7 \text{ g/cm}^3$   
 $m = \underline{\hspace{2cm}} \text{ kg}$



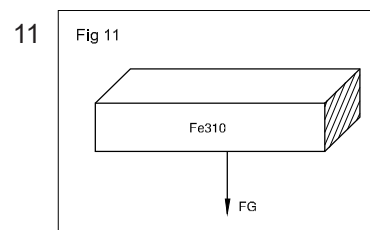
$V = 320 \text{ cm}^3$   
 $\rho = 8.9 \text{ g/cm}^3$   
 $g = 9.80665 \text{ metre/sec}^2$   
 $m = \underline{\hspace{2cm}} \text{ kg}$   
 $FG = \underline{\hspace{2cm}} \text{ N}$



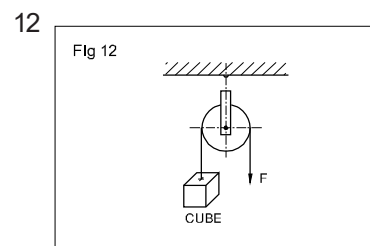
Capacity = 35 litres  
 $g = 10 \text{ metres/sec}^2$   
 $FG = \underline{\hspace{2cm}} \text{ N}$



$(m_1)$  mass of chain = 150 kg  
 Total FG = 8 KN  
 Load =  $\underline{\hspace{2cm}}$  N  
 mass  $m_2 = \underline{\hspace{2cm}}$  kg



$W (FG) = 22.5 \text{ N}$   
 $V (\text{volume}) = \underline{\hspace{2cm}}$



$F = 250 \text{ d N}$   
 side of cube =  $\underline{\hspace{2cm}}$  mm  
 (cubical counter weight balances 'F')



Mensuration - Area and perimeter of square, rectangle and parallelogram

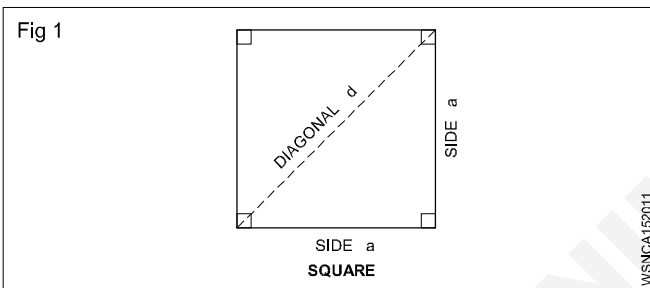
In Engineering field, an Engineer has to estimate the material, manpower, machinery, etc. required to prepare the geometrical objects. Hence we must be very conversant with all relevant formulae connected with geometrical objects.

- Length - l unit
- Breadth or width - b unit
- Diagonal - d unit
- Diameter - d unit

- Radius - r unit
- Semi perimeter - S unit
- Perimeter - P unit
- Circumference - C unit
- Area - A unit<sup>2</sup>
- Total surface area - T.S.A unit<sup>2</sup>
- Lateral surface area - L.S.A unit<sup>2</sup>
- Volume - V unit<sup>3</sup>

**Square**

This is also a four sided figure, opposite sides are parallel. All the four sides are equal. Angle between adjacent side are 90°.



$$A = a^2 \text{ (or) unit}^2$$

$$P = 4a \text{ unit}$$

$$d = \sqrt{2} a \text{ unit}$$

$$a = \frac{d}{\sqrt{2}} \text{ unit where } \sqrt{2} = 1.414$$

Find the area of a brass sheet in the form of a square whose perimeter is 31.2 cm.

$$\text{Perimeter}(P) = 4a = 31.2 \text{ cm}$$

$$\therefore a = \frac{31.2}{4} = 7.8 \text{ cm}$$

$$\text{Area}(A) = a^2$$

$$= 7.8 \times 7.8 = 60.84 \text{ cm}^2$$

**Examples**

1 Find out the circumference, diagonal and area of a square, whose side is 18 cm.

$$\text{Side of the square}(a) = 18 \text{ cm}$$

$$\text{Perimeter}(P) = 4a$$

$$= 4 \times 18 = 72 \text{ cm}$$

$$\text{Diagonal}(d) = \sqrt{2} \times a$$

$$= \sqrt{2} \times 18 = 1.414 \times 18$$

$$= 25.45 \text{ cm}$$

$$\text{Area}(A) = a^2$$

$$= 18 \times 18 = 324 \text{ cm}^2$$

**Perimeter of square = 72 cm**

**Diagonal = 25.45 cm ; Area = 324 cm<sup>2</sup>**

2 If the diagonal of a square measure 10 cm. Find area of the square.

$$\text{Diagonal of the square}(d) = \sqrt{2} a = 10 \text{ cm}$$

$$\text{Side}(a) = \frac{d}{\sqrt{2}}$$

$$\text{Area}(a^2) = \frac{d}{\sqrt{2}} \times \frac{d}{\sqrt{2}} = \frac{d^2}{2}$$

$$= \frac{10^2}{2} = \frac{100}{2}$$

$$= 50 \text{ cm}^2$$

**Area of the square = 50 cm<sup>2</sup>**

3 The perimeter of one square is 748 cm and that of Another is 336 cm. Find the perimeter of a square and total area of the two squares.

$$\text{Side of the square}(a) = \frac{\text{Perimeter}}{4}$$

**1<sup>st</sup> square**

$$\text{Side}(a) = \frac{\text{Perimeter of 1<sup>st</sup> square}}{4}$$

$$= \frac{748}{4} = 187 \text{ cm}$$

**Area(A)**

$$= a^2$$

$$= 187 \times 187$$

$$= 34,969 \text{ cm}^2$$

**2<sup>nd</sup> square**

$$\text{Side}(a) = \frac{\text{Perimeter of 2<sup>nd</sup> square}}{4}$$

$$= \frac{336}{4} = 84 \text{ cm}$$

$$\begin{aligned} \text{Area (A)} &= a^2 \\ &= 84 \times 84 \\ &= 7,056 \text{ cm}^2 \end{aligned}$$

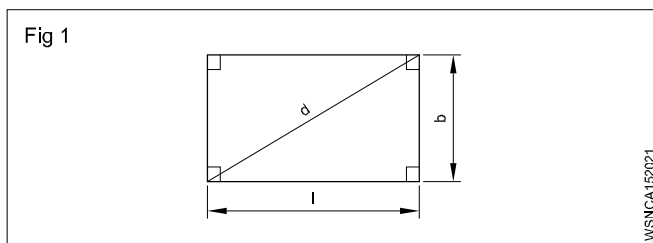
$$\begin{aligned} \text{Total area of two squares} &= 34,969 + 7,056 \\ &= 42,025 \text{ cm}^2 \end{aligned}$$

## Assignment

- Find the Area, Perimeter and diagonal of a square steel plate whose side measures 28.1 cm.
- Find the area of a square whose diagonal is equal to 8.5 cm.
- Find the area of the square if the side of the square is 28 cm.
- Find its side if the area of the square field is 169 m<sup>2</sup>.
- Find the area of the square if the diagonal of the square is 20 cm.
- Find the perimeter of a square whose diagonal is 144 m.
- Find the area if the perimeter of a square plot is 48 m.

## Rectangle

This is a four sided figure. Opposite sides are parallel. Angles between adjacent sides are 90°.



$$A = \text{Area} = \text{length} \times \text{breadth} = l \cdot b \cdot \text{unit}^2$$

$$P = \text{Perimeter} = 2(l + b) \text{ unit}$$

$$\text{Diagonal} = \sqrt{l^2 + b^2} \text{ unit}$$

### Examples

- Find the Area, Perimeter and diagonal of a rectangle whose length and breadth are 144 mm and 60 mm respectively.

$$\begin{aligned} \text{Area} = A &= l \times b \text{ unit}^2 \\ &= 144 \times 60 = 8640 \text{ mm}^2 \end{aligned}$$

$$\begin{aligned} \text{Perimeter} = P &= 2(l + b) \text{ unit} \\ &= 2(144 + 60) \\ &= 2 \times 204 = 408 \text{ mm} \end{aligned}$$

$$\begin{aligned} \text{Diagonal} = d &= \sqrt{l^2 + b^2} \text{ unit} \\ &= \sqrt{144^2 + 60^2} \\ &= \sqrt{20736 + 3600} \\ &= \sqrt{24336} = 156 \text{ mm} \end{aligned}$$

- The perimeter of a rectangle is equal to 42 cm. If its breadth is 9 cm. Find the length of the rectangle.

$$\text{Perimeter} = 42 \text{ cm}$$

$$\text{Breath} = 9 \text{ cm}$$

$$\text{Length} = ?$$

$$\text{Perimeter} = P = 2(l + b)$$

$$2(l + 9) = 42$$

$$l + 9 = 42 \div 2$$

$$l + 9 = 21$$

$$l = 21 - 9$$

$$l = 12 \text{ cm}$$

- The perimeter of a rectangle is 48 cm and its length is 4 cm more than its width. Find the length and breadth of the rectangle.

$$\text{Perimeter (P)} = 48 \text{ cm}$$

$$\text{Breath (b)} = x$$

$$\text{Length (l)} = x + 4$$

$$2(l + b) = \text{Perimeter}$$

$$2(x + 4 + x) = 48$$

$$2(2x + 4) = 48$$

$$4x + 8 = 48$$

$$4x = 48 - 8$$

$$x = \frac{40}{4} = 10$$

$$x = \text{breadth} = 10 \text{ cm}$$

$$\text{length} = x + 4 = 10 + 4 = 14 \text{ cm}$$



- 4 How many rectangular pieces of 50 cm x 20 cm can be cut out from a sheet of 1000 cm x 500 cm.

Sheet size = 1000 cm x 500 cm

Size of the rectangular piece to be cut = 50 cm x 20 cm

$$\text{No. of pieces to be cut in lengthwise} = \frac{1000}{50} = 20$$

$$\text{No. of pieces to be cut in breadthwise} = \frac{500}{20} = 25$$

$$\begin{aligned} \text{Total no. of pieces to be cut out} &= 20 \times 25 \\ &= \mathbf{500} \end{aligned}$$

- 5 The perimeter of a rectangle is 320 metre. Its sides are in the ratio of 5:3. Find the area of the rectangle.

$$\text{Ratio} = 5:3 = l : b$$

$$\text{length } l = 5x$$

$$\text{breadth } b = 3x$$

$$2(l + b) = \text{Perimeter}$$

$$2(5x + 3x) = 320$$

$$2(8x) = 320$$

$$16x = 320$$

$$x = \frac{320}{16} = 20$$

$$l = 5x = 5 \times 20 = 100 \text{ m}$$

$$b = 3x = 3 \times 20 = 60 \text{ m}$$

$$\begin{aligned} \text{Area} &= l \times b \quad (\text{length} = 100\text{m, breath} = 60\text{m}) \\ &= 100 \times 60 \end{aligned}$$

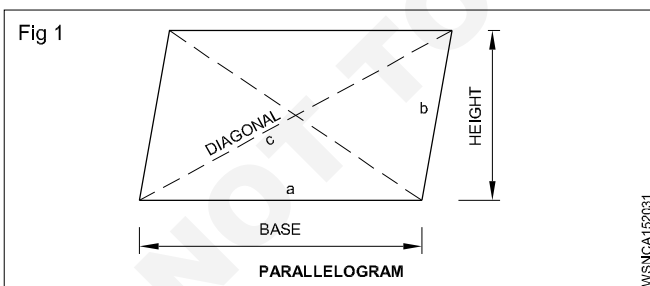
$$\text{Area} = 6000 \text{ m}^2$$

## Assignment

- Find the area of a rectangular plot whose sides are 24 metres and 20 metres respectively. Also find the perimeter of the plot.
- How many rectangular pieces of 5 cm x 4 cm will you get out of 65 cm x 30 cm brass sheet?
- Find its breadth and area if the perimeter of a rectangle is 400 metre and its length is 140 m.
- Find its area, if the opposite sides of a rectangle are 64 cm and 25 cm respectively.
- What is the width of the rectangle if a rectangle has an area of 224 cm<sup>2</sup> and length 16 cm.
- What is the length of the diagonal of a rectangle with sides 16 cm and 12 cm?
- Find the area of the rectangle if the perimeter of the rectangle is 100 cm and the ratio of its length and breadth is 3:2.

## Parallelogram

This is also a four sided figure, opposite side being parallel to each other.



Area of parallelogram = base x height

$$\text{or } = 2x\sqrt{s(s-a)(s-b)(s-c)}$$

Where

$$s = \frac{a+b+c}{2}$$

a and b are adjacent sides.

$$P = 2(a+b)$$

### Examples

- 1 The base and height of a parallelogram are 7.1 cm and 2.85 cm. Calculate its area.

$$\begin{aligned} A &= \text{base} \times \text{height units}^2 \\ &= 7.1 \times 2.85 \\ &= \mathbf{20.235 \text{ cm}^2} \end{aligned}$$

- 2 Find the height of a parallelogram whose area is 20 cm<sup>2</sup> and base is 10 cm.

$$\begin{aligned} A &= \text{base} \times \text{height units}^2 \\ h &= \frac{\text{area}}{\text{base}} \\ &= \frac{20}{10} \\ &= \mathbf{2 \text{ cm}} \end{aligned}$$

3. Two sides of a parallelogram are 12 cm and 8 cm. The diagonal is 10 cm long. Find the area of the parallelogram.

$$\begin{aligned} \text{Area } A &= 2x\sqrt{s(s-a)(s-b)(s-c)} \text{ units}^2 \\ s &= \frac{a+b+c}{2} \\ &= \frac{12+8+10}{2} \\ &= \frac{30}{2} \\ &= 15 \end{aligned}$$

$$\begin{aligned} A &= 2x\sqrt{15(15-12)(15-8)(15-10)} \\ &= 2x\sqrt{15 \times 3 \times 7 \times 5} \\ &= 2x\sqrt{1575} \\ &= 2 \times 39.686 \\ \text{Area } A &= 79.37 \text{ cm}^2 \end{aligned}$$

## Assignment

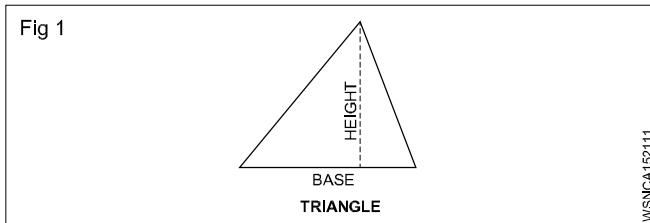
- 1 Find the area of a parallelogram, if its base and height are 8.1 cm and 30.8 cm respectively.
- 2 Find the area of a parallelogram, if the sides of a field in the shape of parallelogram are 12 m and 17 m and one of the diagonal is 25 m.
- 3 Find the base of a parallelogram whose height is 12 cm and area is 120 cm<sup>2</sup>.
- 4 Find the height of a parallelogram whose base is 40 cm and area is 320 cm<sup>2</sup>.
- 5 Find the area of the land if the sides of a land in the shape of a parallelogram are 24 m and 28 m respectively and one of the diagonal is 30 m.
- 6 What is the perimeter of parallelogram if base is 10 cm and other side is 5 cm?
- 7 Find the area of parallelogram if its base and height are 25 cm and 12 cm.
- 8 Find the base of a parallelogram if height is 15 cm and area is 150 cm<sup>2</sup>.
- 9 Find the area of parallelogram if side is 5 cm, diagonal is 8 cm and diagonal bisects each other at right angles.
- 10 Find the height of a parallelogram if base is 80 cm and area is 640 cm<sup>2</sup>.
- 11 Find the area of parallelogram if its base and height are 15 cm and 8 cm.
- 12 Calculate the perimeter and area of parallelogram if base, height are 12.7 cm, 5.5 cm and other side is 6.5 cm.
- 13 Find the height of parallelogram if the area is 20 cm<sup>2</sup> and base is 10 cm.

Mensuration - Area and perimeter of triangles

Triangles

Tri means three. Hence tri- angle means three angled figure. For construction of three angled figure, there should be three sides. Hence triangle means three sided figure. Sum of the three angles of any triangle = 180°.

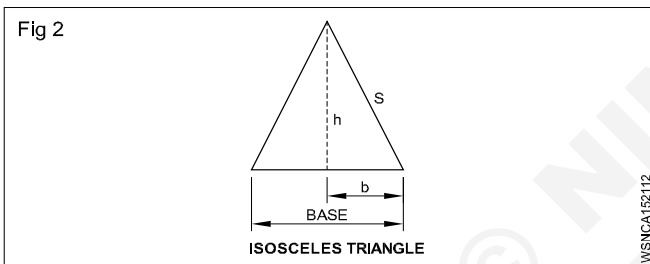
i Any triangle.



Area of any triangle =  $\frac{1}{2}$  x Base x Height unit<sup>2</sup>

ii Isosceles Triangle

In this triangle two of its sides are equal.



Area of isosceles triangle =  $\frac{1}{2}$  x Base x Height

Where

base = 2.b

s = One of equal sides (or) Slant height

h = Height =  $\sqrt{s^2 - b^2}$

Area of isosceles triangle =  $\frac{1}{2}$  x 2b x  $\sqrt{s^2 - b^2}$   
 = b .  $\sqrt{s^2 - b^2}$  unit<sup>2</sup>

(Where b= half of base)

(or) Area of Isosceles triangle =  $\frac{1}{4}$  b  $\sqrt{4a^2 - b^2}$  unit<sup>2</sup>

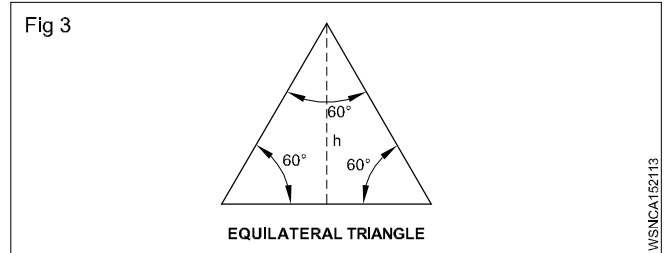
a = Equal sides

b = Base

iii Equilateral triangle

In this triangle all the three sides are equal. Hence angle between adjacent sides is 60° (because no. of angles)

total = 180° ; angle between sides =  $\frac{180}{3} = 60^\circ$



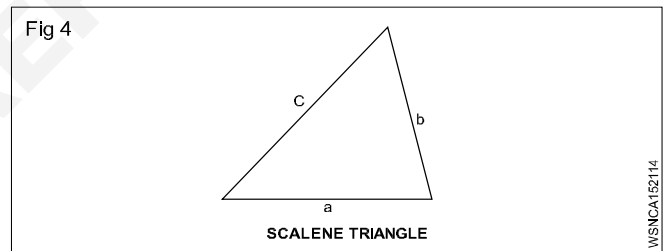
Area of equilateral triangle =  $\frac{\sqrt{3}}{4}$  x side<sup>2</sup>  
 =  $\frac{\sqrt{3}}{4}$  x a<sup>2</sup> unit<sup>2</sup>

Where  $\sqrt{3} = 1.732$   
 Perimeter P = 3a unit

P =  $\frac{\sqrt{3}}{2}$  a unit

iv Scalene triangle

In this triangle the sides are not equal. Angles between the sides, are also not equal. we may also call this triangle as irregular triangle.



Area of triangle =  $\sqrt{s(s-a)(s-b)(s-c)}$  unit<sup>2</sup>

where

a,b,c are sides of triangle

s = Semi perimeter =  $\frac{a+b+c}{2}$  unit

v Right angled triangle

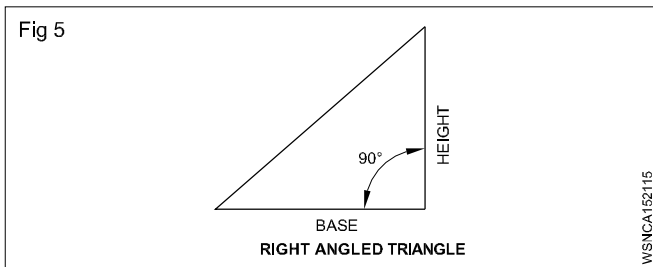
In this triangle, angle between one of two adjacent sides is 90°. Right angle means ninety degrees. That's why right angled triangle means, one of the angles of this triangle is definitely ninety degrees.

Area of right angled triangle

=  $\frac{1}{2}$  x Base x Height

=  $\frac{1}{2}$  bh unit<sup>2</sup>

Hypotenuse =  $\sqrt{\text{Base}^2 + \text{Height}^2}$



Where hypotenuse means, the diagonal or largest length of the side of right angled triangle.

### Examples

- 1 Calculate its area if the base and height of a Right angled triangle are 10 cm and 3.5 cm respectively.

Base (b) = 10 cm

Height (h) = 3.5 cm

Area (A) = ?

$$A = \frac{1}{2} \times b \times h$$

$$= \frac{1}{2} \times 10 \times 3.5$$

$$= 17.5 \text{ cm}^2$$

- 2 Calculate the base of a triangle having an area of 15 cm<sup>2</sup> and height is 3.5 cm.

Area (A) = 15 cm<sup>2</sup>

Height (h) = 3.5 cm

Base (b) = ?

$$\frac{1}{2} \times b \times h = A$$

$$\frac{1}{2} \times b \times 3.5 = 15$$

$$b = \frac{15 \times 2}{3.5}$$

$$= 8.57 \text{ cm}$$

- 3 Calculate the height of a triangle whose area is 60 cm<sup>2</sup> and base is 10 cm.

Area (A) = 60 cm<sup>2</sup>

Base (B) = 10 cm

Height (h) = ?

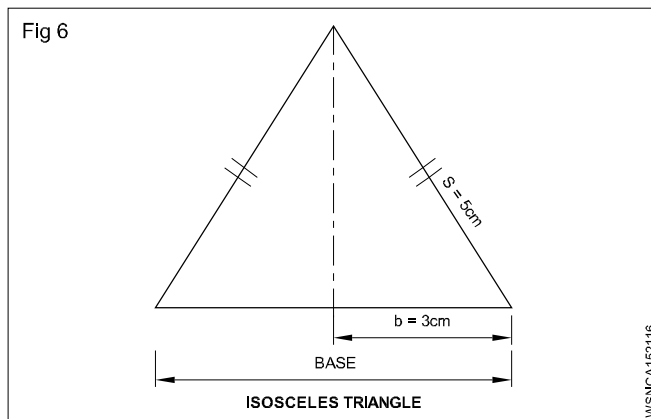
$$\frac{1}{2} \times b \times h = A$$

$$\frac{1}{2} \times 10 \times h = 60$$

$$h = \frac{60 \times 2}{10}$$

height h = 12 cm

- 4 Find the area of an isosceles triangle whose base is 6 cm long and each of the other two sides 5 cm long.



Base (b) = 6 cm = 3 cm

Equal sides or slant height 's' = 5 cm

Area (A) = ?

$$A = b \times \sqrt{s^2 - b^2}$$

$$= 3 \times \sqrt{5^2 - 3^2}$$

$$= 3 \times \sqrt{25 - 9}$$

$$= 3 \times \sqrt{16}$$

$$= 3 \times 4$$

$$= 12 \text{ cm}^2$$

or

$$A = \frac{1}{4} b \sqrt{4a^2 - b^2}$$

$$= \frac{1}{4} \times 6 \sqrt{4 \times 5^2 - 6^2}$$

$$= \frac{1}{4} \times 6 \times 8$$

$$= 12 \text{ cm}^2$$

- 5 Find its height if an isosceles triangle has base of 200 mm and its area is 2000 mm<sup>2</sup>.

Base = 200 mm

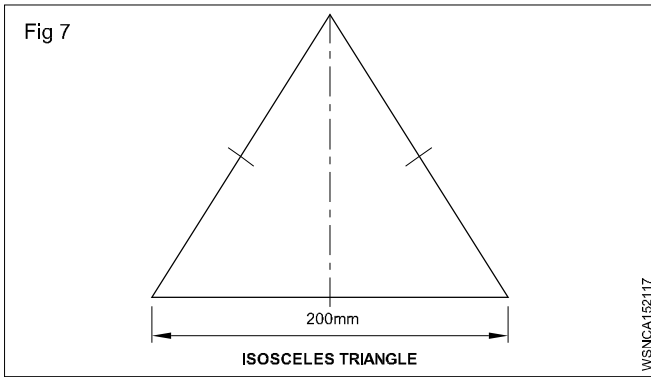
Area = 2000 mm<sup>2</sup>

h = ?

$$\frac{1}{2} \times b \times h = A$$

$$\frac{1}{2} \times 200 \times h = 2000$$

$$h = \frac{2000 \times 2}{200} = 20 \text{ mm}$$



- 6 Find the area of an equilateral triangle whose side is 5 cm.

$$\begin{aligned} \text{Area} &= \frac{\sqrt{3}}{4} a^2 \text{ unit}^2 \\ &= \frac{1.732}{4} \times 5 \times 5 \\ &= \mathbf{10.825 \text{ cm}^2} \end{aligned}$$

- 7 Calculate its perimeter if one side of an equilateral triangle is 55 mm long.

$$\begin{aligned} \text{Side} &= 55 \text{ mm} \\ \text{Perimeter (P)} &= ? \\ P &= 3a \text{ unit} \\ &= 3 \times 55 \\ &= \mathbf{165 \text{ mm}} \end{aligned}$$

- 8 Find the area of the triangle having its sides are 9cm, 10cm and 12 cm.

$$\begin{aligned} \text{Semi Perimeter} &= \frac{a+b+c}{2} \text{ unit} \\ &= \frac{9+10+12}{2} = \frac{31}{2} \\ &= 15.5 \text{ cm} \\ \text{Area A} &= \sqrt{s(s-a)(s-b)(s-c)} \text{ unit}^2 \\ &= \sqrt{15.5(15.5-9)(15.5-10)(15.5-12)} \\ &= \sqrt{15.5 \times 6.5 \times 5.5 \times 3.5} \\ &= \sqrt{1939.4375} \\ &= \mathbf{44.03 \text{ cm}^2} \end{aligned}$$

- 9 Find the cost of polishing on both sides of a triangular metal plate has sides 60 cm, 50 cm and 20 cm at the rate of Rs.1.35 per 100 cm<sup>2</sup>

$$\text{Semi Perimeter} = \frac{a+b+c}{2} \text{ unit}$$

$$= \frac{60+50+20}{2} = \frac{130}{2}$$

$$= 65 \text{ cm}$$

$$\text{Area A} = \sqrt{s(s-a)(s-b)(s-c)} \text{ unit}^2$$

$$= \sqrt{65(65-60)(65-50)(65-20)}$$

$$= \sqrt{65 \times 5 \times 15 \times 45}$$

$$= \mathbf{468.4 \text{ cm}^2}$$

$$\text{Area of polish on both sides} = 2 \times 468.4$$

$$= 936.8 \text{ cm}^2$$

$$\text{Cost of polish per 100 cm}^2 = \text{Rs. } 1.35$$

$$\therefore \text{Cost of polish is } 936.8 \text{ cm}^2 = \frac{936.8}{100} \times 1.35$$

$$= \text{Rs. } 12.65$$

- 10 Find the area of the right angled triangle with base 20 cm and height 8 cm.

$$\text{Base } b = 20 \text{ cm}$$

$$\text{Equal sides or slant height} = 8 \text{ cm}$$

$$\text{Area (A)} = ?$$

$$\text{Area (A)} = \frac{1}{2} \times \text{base} \times \text{height unit}^2$$

$$= \frac{1}{2} \times 20 \times 8$$

$$= \mathbf{80 \text{ cm}^2}$$

- 11 Find the area of the right angled triangle if the sides containing the right angle being 10.5 cm and 8.2 cm.

$$\text{Area (A)} = \frac{1}{2} \times \text{base} \times \text{height unit}^2$$

$$= \frac{1}{2} \times 10.5 \times 8.2$$

$$= \mathbf{43.05 \text{ cm}^2}$$

- 12 Calculate the perpendicular height of the triangle if the area of the right angled triangle is 19.44 m<sup>2</sup> and its one of the adjacent side containing the right angle being 5.4 m.

$$\frac{1}{2} \times \text{base} \times \text{height unit}^2 = \text{Area}$$

$$\frac{1}{2} \times 5.4 \times h = 19.44$$

$$h = \frac{19.44 \times 2}{5.4}$$

$$= 7.2 \text{ m}$$

13 Calculate the base of a right angled triangle having an area of 15 cm<sup>2</sup>. If its height is 3.5 cm.

$$\frac{1}{2} \times \text{base} \times \text{height unit}^2 = \text{Area}$$

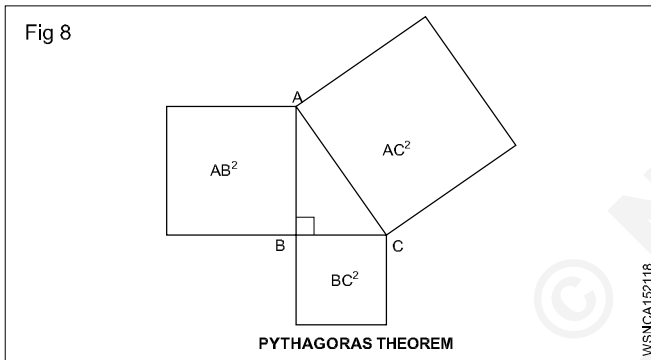
$$\frac{1}{2} \times b \times 3.5 = 15$$

$$b = \frac{15 \times 2}{3.5}$$

$$= 8.57 \text{ cm}$$

### Pythagoras theorem

In a right angled triangle the area of the square drawn with the hypotenuse as the side is equal to the sum of the areas of the squares drawn with the other two sides.



$$\angle B = 90^\circ$$

AC = Hypotenuse

AB & BC = Adjacent sides

As per pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

$$\therefore AC = \sqrt{AB^2 + BC^2}$$

1 Calculate the hypotenuse of a right angled triangle whose base is 5 cm and height is 12 cm.

As per pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

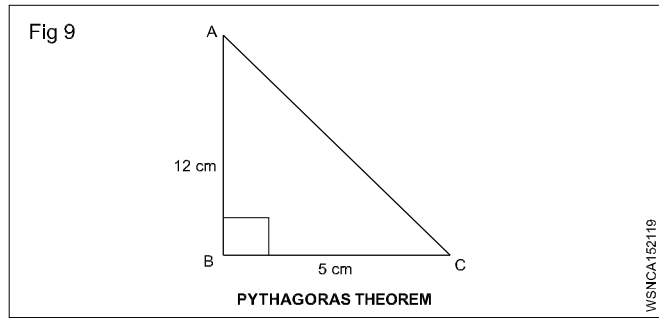
$$= 12^2 + 5^2$$

$$= 144 + 25$$

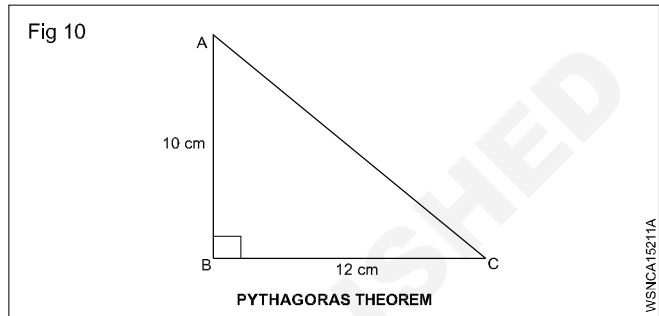
$$= 169$$

$$AC = \sqrt{169}$$

$$= 13 \text{ cm}$$



2 What is the length of the hypotenuse of a right angled triangle, when the sides containing the right angles are 10 cm and 12 cm.



As per pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

$$= 10^2 + 12^2$$

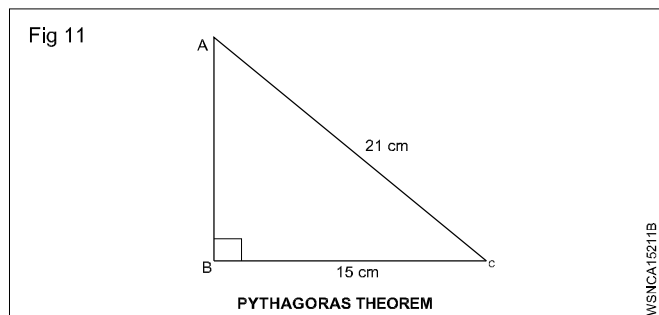
$$= 100 + 144$$

$$= 244$$

$$AC = \sqrt{244}$$

$$= 15.62 \text{ cm}$$

3 Find the height of a right angled triangle whose base is 15 cm and hypotenuse is 21 cm.



As per pythagoras theorem,

$$AB^2 + BC^2 = AC^2$$

$$AB^2 + 15^2 = 21^2$$

$$AB^2 = 441 - 225$$

$$= 216$$

$$AB = \sqrt{216}$$

$$= 14.7 \text{ cm}$$

Mensuration - Area and perimeter of circle, semi-circle, circular ring, sector of circle, hexagon and ellipse

Circle

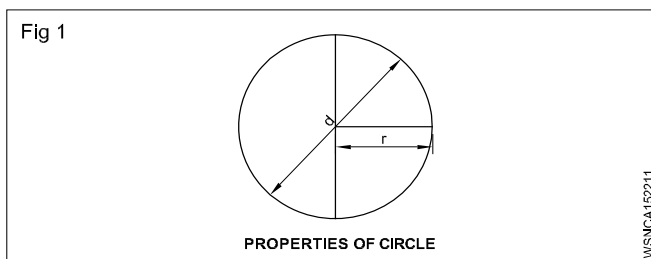
It is the path of a point which is always equal from its centre is called a circle.

r = radius of the circle

d = diameter of the circle

$$\pi = \frac{22}{7} = 3.14$$

Area of the circle =  $\pi r^2$



(or) 
$$= \frac{\pi}{4} d^2 \text{ unit}^2$$

Circumference of the circle  $2\pi r$  (or)  $\pi d$  unit

Examples

1 Find the area of a circle whose radius is 1.54 m. Also find its circumference.

radius r = 1.54 cm

Area A = ?

Circumference C = ?

$$A = \pi r^2 \text{ unit}^2$$

$$= \frac{22}{7} \times 1.54 \times 1.54$$

$$= 7.4536 \text{ m}^2$$

$$C = 2\pi r \text{ unit}$$

$$= 2 \times \frac{22}{7} \times 1.54$$

$$= 9.68 \text{ m}$$

2 Find out the circumference if the area of a circular shape of land is 616 m<sup>2</sup>.

A =  $\pi r^2$  unit<sup>2</sup>

$$r^2 = \frac{616}{\pi}$$

$$= \frac{616 \times 7}{22}$$

$$= 196$$

$$r = \sqrt{196}$$

$$= 14 \text{ m}$$

Circumference =  $2\pi r$  unit

$$= 2 \times \frac{22}{7} \times 14$$

$$= 88 \text{ m}$$

3 Find the side of square into which it can be bent if a wire is in the form of a circle of radius 49 cm.

radius of circle r = 49 cm

side of square = ?

Perimeter of the square = Perimeter of the circle

$$4a = 2\pi r$$

$$4a = 2 \times \frac{22}{7} \times 49$$

$$4a = 308$$

$$a = \frac{308}{4}$$

$$= 77 \text{ cm}$$

4 Find its radius if the difference between the circumference and diameter of a circle is 28 cm.

Circumference - Diameter = 28 cm

$$2\pi r - d = 28$$

$$2\pi r - 2r = 28$$

$$2r(\pi - 1) = 28$$

$$2r \left( \frac{22}{7} - 1 \right) = 28$$

$$2r \left( \frac{22-7}{7} \right) = 28$$

$$2r \times \frac{15}{7} = 28$$

$$r = \frac{28 \times 7}{15 \times 2}$$

$$= 6.53 \text{ cm}$$

- 5 What is the side of the largest square cut out from a circle of 50 cm dia.?

Diagonal of a square = Diameter of the circle

$$\sqrt{2}a = 50$$

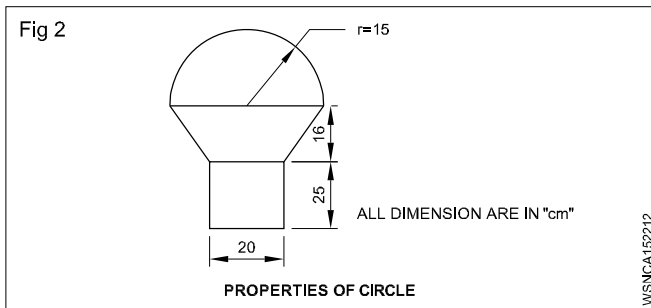
$$a = \frac{50}{\sqrt{2}}$$

$$= \frac{50}{1.414}$$

$$= \mathbf{35.36 \text{ cm}}$$

- 6 Calculate the area of the figure given below.

Area of rectangle = lb unit<sup>2</sup>



$$= 25 \times 20 \text{ cm}^2$$

$$= 500 \text{ cm}^2$$

$$\text{Area of Trapezium} = \frac{1}{2} \times (a + b) h$$

$$= \frac{1}{2} \times (30 + 20) 16 \text{ cm}^2$$

## Semi circle

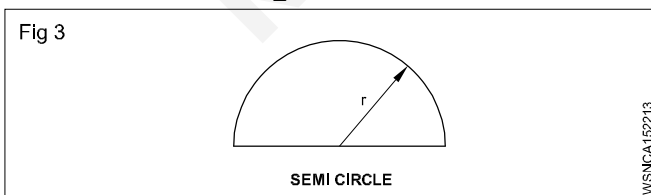
A semi circle is a sector whose central angle is 180°.

Length of arc of semi circle.

$$\text{Length of arc } l = 2\pi r \times \frac{180}{360}$$

$$= 2\pi r \times \frac{1}{2} = \pi r \text{ unit}$$

$$\text{Area of semi circle} = \frac{\pi r^2}{2} \text{ Sq. units}$$



$$\text{Perimeter of a semi circle} = \frac{2\pi r}{2} + 2r$$

$$= \pi r + 2r$$

$$= r(\pi + 2) \text{ unit}$$

$$= \frac{1}{2} \times 50 \times 16 \text{ cm}^2$$

$$= 400 \text{ cm}^2$$

$$\text{Area of Semi circle} = \frac{\pi r^2}{2} \text{ unit}^2$$

$$= \pi \times 15^2 \times \frac{1}{2} \text{ cm}^2$$

$$= 353.57 \text{ cm}^2$$

$$\text{Total area of the figure} = 500 + 400 + 353.57$$

$$= \mathbf{1253.57 \text{ cm}^2}$$

- 7 Find the area of remaining steel plate if in a rectangular steel plate 16 cm x 12 cm, there are 6 holes each 4 cm in diameter.

Area of a rectangular plate = length x breadth unit<sup>2</sup>

$$= 16 \times 12$$

$$= 192 \text{ cm}^2$$

$$\text{No. of holes} = 6$$

$$\text{Radius of hole} = 2 \text{ cm}$$

$$\text{Area of 6 holes} = 6 \times \pi r^2 \text{ unit}^2$$

$$= 6 \times \frac{22}{7} \times 2 \times 2 \text{ unit}^2$$

$$= 75.43 \text{ cm}^2$$

$$\text{Area of remaining plate} = 192 - 75.43$$

$$= \mathbf{116.57 \text{ cm}^2}$$

## Examples

- 1 Calculate the circumference and area of a semi circle whose radius is 6 cm.

$$\text{radius } r = 6 \text{ cm}$$

$$\text{Area } A = ?$$

$$\text{Circumference } c = ?$$

$$A = \frac{\pi r^2}{2} \text{ unit}^2$$

$$= \frac{22}{7} \times \frac{1}{2} \times 6^2$$

$$\text{Area (A)} = \frac{22}{7} \times \frac{1}{2} \times 36$$

$$= \frac{396}{7} = 56.57 \text{ cm}^2$$

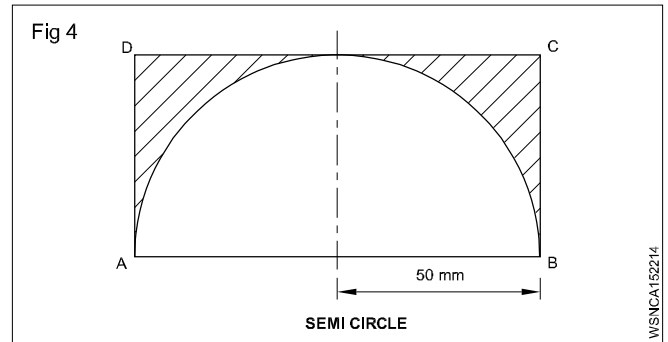
$$\text{Perimeter of a semicircle} = 6\left(\frac{22}{7} \times 2\right)$$



$$\begin{aligned}
 &= 6 \left( \frac{22 + 14}{7} \right) \\
 &= 6 \times \frac{36}{7} \\
 &= \frac{216}{7} \\
 &= \mathbf{30.86 \text{ cm}}
 \end{aligned}$$

- 2 From the figure given below ABCD is a steel plate, a semi circular plate of radius 50 mm has been prepared by gas cutting. Find the waste area.

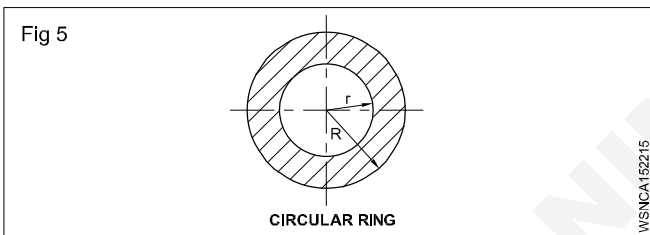
Plate length AB = 100 mm  
 Breadth BC = 50 mm  
 Radius = 50 mm



Waste area = Plate area - Area of semi circle

$$\begin{aligned}
 &= lb - \frac{\pi r^2}{2} \\
 &= 100 \times 50 - \frac{22 \times 50 \times 50}{7 \times 2} \\
 &= 5000 - 3928.57 \\
 &= \mathbf{1071.43 \text{ mm}^2}
 \end{aligned}$$

## Circular ring



R = Outer radius of circular ring

r = Inner radius of circular ring

Area of circular ring =  $\pi (R^2 - r^2)$  unit<sup>2</sup>  
 or

$A = \pi (R + r) (R - r)$  unit<sup>2</sup>

- 1 Calculate the area of cross section of pipe having outside dia of 17 cm and inside dia of 14 cm.

Given:

Outer dia of pipe = 17 cm

Outer radius of pipe (R) =  $\frac{17}{2} = 8.5$  cm

Inner dia of pipe = 14 cm

Inner radius of pipe (r) =  $\frac{14}{2} = 7$  cm

To find:

Area of cross section of pipe = ?

Solution:

$$\begin{aligned}
 \text{Area of cross section of pipe} &= \pi (R + r) (R - r) \text{ unit}^2 \\
 &= \pi (8.5 + 7) (8.5 - 7) \\
 &= \frac{22}{7} \times 15.5 \times 1.5 \text{ cm}^2 \\
 &= \mathbf{73 \text{ cm}^2}
 \end{aligned}$$

- 2 Find the distance between the boundaries and the area of the circular ring, if the circumference of two concentric circle are 134 cm and 90 cm.

Given:

Circumference of outer circle = 134 cm

Circumference of inner circle = 90 cm

To find:

Distance between the circles = ?

Area of circular ring = ?

Solution:

Circumference of outer circle = 134 cm

$$2\pi R = 134 \text{ cm}$$

$$R = \frac{134}{2\pi} = 21.32 \text{ cm}$$

Circumference of inner circle = 90 cm

$$2\pi r = 90 \text{ cm}$$

$$r = \frac{90}{2\pi} = 14.32 \text{ cm}$$

Distance between the circle = R - r

$$= 21.32 - 14.32 \text{ cm}$$

$$= 7 \text{ cm}$$

Area of circular ring =  $\pi (R + r) (R - r)$  unit<sup>2</sup>

$$= \pi (21.32 + 14.32) (21.32 - 14.32) \text{ cm}^2$$

$$= \frac{22}{7} \times 35.64 \times 7 \text{ cm}^2$$

$$= \mathbf{784.08 \text{ cm}^2}$$

3 A wire can be bend in the form of a circle of radius 56 cm. If it is bend in a form of a square, find the side.

Given:

$$\text{Radius of circle} = 56 \text{ cm}$$

To find:

$$\text{Side of square} = ?$$

Solution:

$$\text{Radius of circle} = 56 \text{ cm}$$

$$\text{Circumference of circle} = 2\pi r \text{ unit} = 2\pi \times 56 \text{ cm}$$

$$\text{Side of square} = x \text{ cm}$$

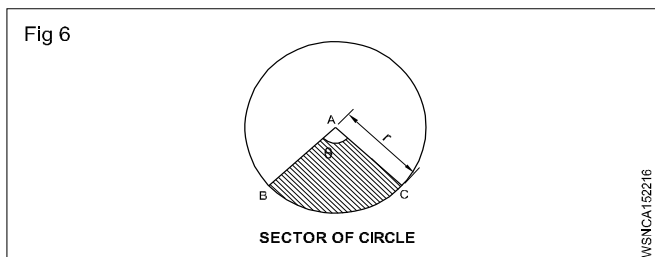
Wire can be bend from the form of round to square

$$\text{Perimeter of square} = \text{circumference of circle}$$

$$4 \times a = 352 \text{ cm}$$

$$a = \frac{352}{4} = 88 \text{ cm}$$

## Sector of Circle



$\theta$  = Angle of sector of circle

$l$  = Arc length

$r$  = radius

$$\text{Length of Arc } l = \frac{\theta}{360^\circ} \times 2\pi r \text{ unit}$$

$$\text{Perimeter } P = 2r + l \text{ unit}$$

$$\text{Area} = \frac{\theta}{360^\circ} \times \pi r^2 \text{ unit}^2 \quad (\text{or}) \quad A = \frac{lr}{2} \text{ unit}^2$$

1 Find the perimeter and area of a sector of circle of radius 7 cm and its angle is  $120^\circ$ .

Given:

$$\text{Angle of sector of circle} = 120^\circ$$

$$\text{Radius} = 7 \text{ cm}$$

To find:

$$\text{Perimeter} = ? , \text{Area} = ?$$

Solution:

$$\begin{aligned} \text{Length of arc } (l) &= \frac{\theta}{360^\circ} \times 2\pi r \text{ unit} \\ &= \frac{120}{360} \times 2 \times \frac{22}{7} \times 7 \text{ cm} \\ &= 14.67 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{Perimeter} &= 2r + l \text{ unit} \\ &= 2 \times 7 + 14.67 \text{ cm} \\ &= 28.67 \text{ cm} \end{aligned}$$

$$\text{Area} = \frac{\theta}{360^\circ} \times \pi r^2 \text{ unit}^2$$

$$\text{Area} = \frac{120^\circ}{360^\circ} \times \frac{22}{7} \times 7 \text{ cm}^2 = 51.33 \text{ cm}^2$$

2 Find the radius of the circle if the angle is  $60^\circ$  and the area of a sector of a circle is  $144 \text{ cm}^2$ ,

Given:

$$\text{Area of sector of circle } (A) = 144 \text{ cm}^2$$

$$\text{Angle of sector of circle } \theta = 60^\circ$$

To find:

$$\text{Radius of circle} = ?$$

Solution:

$$\begin{aligned} \text{Area } (A) &= \frac{\theta}{360^\circ} \times \pi r^2 \text{ unit}^2 \\ 144 &= \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times r^2 \text{ cm}^2 \\ r^2 &= 274.91 \text{ cm}^2 \\ r &= \sqrt{274.91} = 16.58 \text{ cm} \end{aligned}$$

3 Find the area of the sector whose angle is  $105^\circ$ , and the perimeter of sector of circle is 18.6 cm.

Given:

$$\text{Perimeter of a sector of a circle} = 18.6 \text{ cm}$$

$$\text{Angle of sector of circle} = 105^\circ$$

To find:

$$\text{Area} = ?$$

Solution:

$$\begin{aligned} \text{Length of Arc } (l) &= \frac{\theta}{360^\circ} \times 2\pi r \text{ unit} \\ l &= \frac{105^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times r \\ &= 1.83r \end{aligned}$$

$$\begin{aligned} \text{Perimeter } (P) &= l + 2r \text{ unit} \\ 18.6 &= 1.83r + 2r \\ 3.83r &= 18.6 \text{ cm} \end{aligned}$$

$$r = \frac{18.6}{3.83} = 4.86 \text{ cm}$$

$$\begin{aligned} \text{Area A} &= \frac{\theta}{360^\circ} \times \pi r^2 \text{ unit}^2 \\ &= \frac{105^\circ}{360^\circ} \times \frac{22}{7} \times (4.86)^2 \text{ cm}^2 \\ &= 21.65 \text{ cm}^2 \end{aligned}$$

4 Find the area, if the radius is 12.4 cm and the perimeter of a sector of a circle is 64.8 cm.

Given:

$$\text{Perimeter P} = 64.8 \text{ cm}$$

$$\text{Radius r} = 12.4 \text{ cm}$$

To find:

$$\text{Area A} = ?$$

Solution:

$$\text{Perimeter P} = l + 2r \text{ unit}$$

$$l = P - 2r \text{ unit}$$

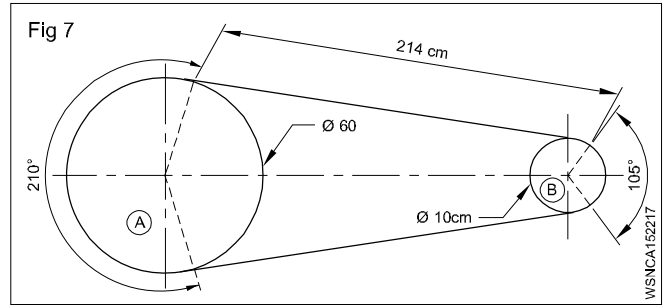
$$= 64.8 - 2(12.4) \text{ cm}$$

$$= 64.8 - 24.8 = 40 \text{ cm}$$

$$\text{Area A} = \frac{lr}{2} \text{ unit}^2 = \frac{40 \times 12.4}{2}$$

$$= 248 \text{ cm}^2$$

5 Find out the length of the belt, if the arrangement of a belt is shown in the figure below.



Solution:

$$\text{Length } l_A = \frac{\theta}{360^\circ} \times 2\pi r \text{ unit}$$

$$= \frac{210^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 30 = 110 \text{ cm}$$

$$\text{Length } l_B = \frac{\theta}{360^\circ} \times 2\pi r \text{ unit}$$

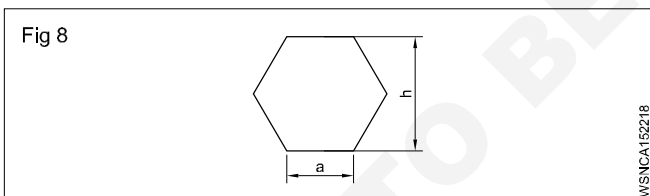
$$= \frac{105^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 5 = 91.7 \text{ cm}$$

$$= l_A + l_B + 2 \times 214 \text{ cm}$$

$$= 110 + 91.7 + 428 \text{ cm}$$

$$= 547.17 \text{ cm}$$

## Hexagon



Side = a unit

Perimeter P = 6a unit

Area A =  $6 \times \frac{\sqrt{3}}{4} \times a^2$  units<sup>2</sup> (Area of 6 equilateral triangle)

DAF (Distance Across Flats) =  $\sqrt{3} \times a$  unit

DAC (Distance Across Corners) = 2 x a unit

1 Find out the perimeter, area, DAF and DAC of a regular hexagon whose side is 2cm.

(DAF - Distance Across Flats)

(DAC - Distance Across Corners)

Given: Side of hexagon (a) = 2cm

To Find: P = ?, A = ?, DAF = ?, DAC = ?

Solution:

Perimeter of hexagon (P) = 6a unit

$$= 6a \text{ unit} = 6 \times 2 \text{ cm} = 12 \text{ cm}$$

Area of hexagon A =  $6 \times \frac{\sqrt{3}}{4} \times a^2$  unit<sup>2</sup>

$$= 6 \times \frac{1.732}{4} \times 2^2$$

$$= 10.392 \text{ cm}^2$$

DAF (Distance Across

Flats) =  $\sqrt{3} \times a$  unit

$$= \sqrt{3} \times 2 = 1.732 \times 2$$

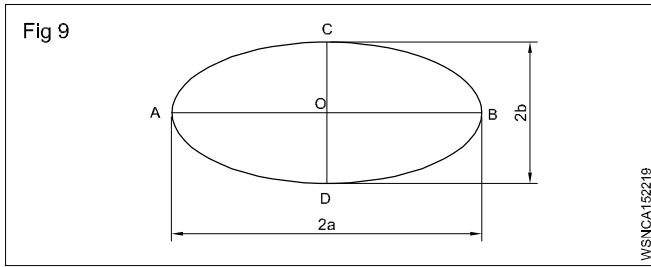
$$= 3.464 \text{ cm}$$

DAC (Distance Across

Corners) = 2 x a unit

$$= 2 \times 2 = 4 \text{ cm}$$

## Ellipse



Major axis  $AB = 2a$

Half of Major axis  $OB = a$ ,

Minor axis  $CD = 2b$

Half of Minor axis  $OC = b$

Area of ellipse  $A = \pi \times a \times b \text{ unit}^2$

Perimeter of ellipse  $P = 2\pi \sqrt{\frac{(a^2 + b^2)}{2}} \text{ unit}$

- Find its area and perimeter, if the major and minor axis of an ellipse are 12 cm and 8 cm respectively.

Solution:

$$\begin{aligned} \text{Major axis } 2a &= 12 \text{ cm} \\ a &= \frac{12}{2} = 6 \text{ cm} \end{aligned}$$

$$\text{Minor axis } 2b = 8 \text{ cm}$$

$$b = \frac{8}{2} = 4 \text{ cm}$$

$$\text{Area } A = \pi \times a \times b \text{ unit}^2$$

$$= \frac{22}{7} \times 6 \times 4 \text{ cm}^2$$

$$= 75.43 \text{ cm}^2$$

$$\text{Perimeter (P)} = 2\pi \sqrt{\frac{(a^2 + b^2)}{2}} \text{ unit}$$

$$= 2 \times \frac{22}{7} \sqrt{\frac{(6^2 + 4^2)}{2}} \text{ unit}$$

$$= 2 \times \frac{22}{7} \sqrt{\frac{36 + 16}{2}} \text{ unit}$$

$$= 2 \times \frac{22}{7} \times \sqrt{26}$$

$$= 2 \times \frac{22}{7} \times 5.1 = 32.06 \text{ cm}$$

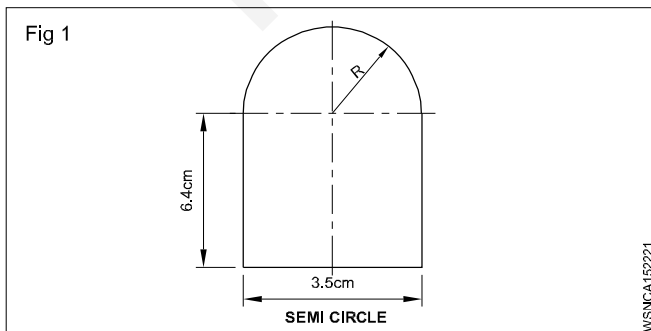
## Assignment

### Circle

- Find the circumference and area of a circle whose radius is 10.00 metre.
- Find its diameter if the area of a circle is  $330 \text{ cm}^2$ .
- Find its area if the circumference of a circle is 50 cm.
- Find out the area and circumference of a circle of diameter is 50 cm.
- Find its area if the circumference of a circle is 44 cm.

### Semi circle

- Calculate the circumference and area of semi circle whose radius is 14 cm.
- Find area of the given figure.



### Circular ring

- Find out area of a ring washer, whose inner radius and outer radius are 13 cm and 15 cm respectively.
- Find the area of a ring portion of a washer whose outer dis is 30 m and inner dis is 20 m. Also calculate the difference between the circumference of circles.

### Sector of circle

- Find the perimeter and area of a sector of a circle of radius 5cm and its angle is  $96^\circ$ .
- Find the radius of the circle if the angle is  $90^\circ$  and the area of sector of a circle is  $196 \text{ cm}^2$ .

### Hexagon

- Find out the Area, perimeter, DAF, and DAC of hexagon of side 4cm.
- Find the area of cross section of a regular hexagon rod whose side is 7.5 cm.

### Ellipse

- Find the area of the biggest ellipse that can be inscribed in a rectangle of length 18 cm and breadth 12 cm. Also calculate its perimeter.
- How much fencing will be required to enclose an elliptical plot of ground the axes of the ellipse being 200 and 170 meter respectively.

Mensuration - Surface area and volume of solids - cube, cuboid, cylinder, sphere and hollow cylinder

**Cube**

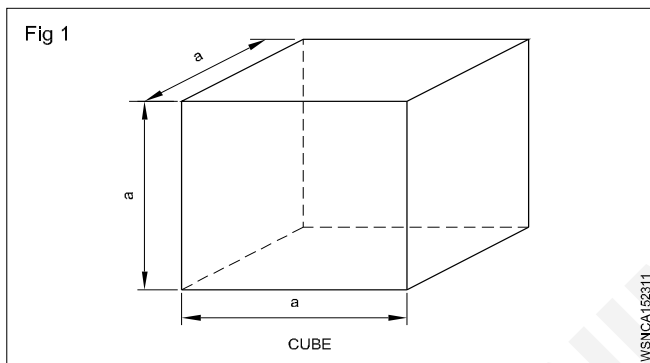
All sides of cube are same i.e length, breadth and height have same value. It is bounded by six equal square faces.

Volume of cube = side x side x side  
 =  $a^3$  unit<sup>3</sup>

Lateral surface area =  $4a^2$  unit<sup>2</sup>

Total surface area = 6 x side x side  
 =  $6a^2$  unit<sup>2</sup>       $\sqrt{3} = 1.732$

Diagonal d =  $\sqrt{3}a$



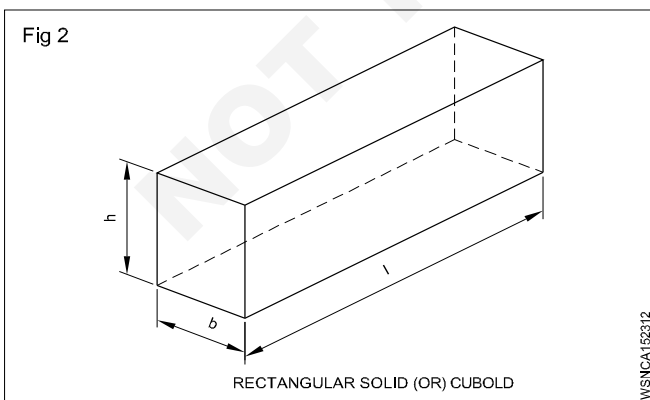
**Rectangular solid (or) cuboid**

Rectangular solid is bounded by six rectangular surfaces and opposite surfaces are equal and parallel to each other.

Volume of rectangular solid = Length x breadth x height  
 =  $l \cdot b \cdot h$  unit<sup>3</sup>

Lateral surface area =  $2h(l+b)$  unit<sup>2</sup>

Total surface area =  $2lb + 2bh + 2hl$   
 =  $2(lb+bh+hl)$  unit<sup>2</sup>

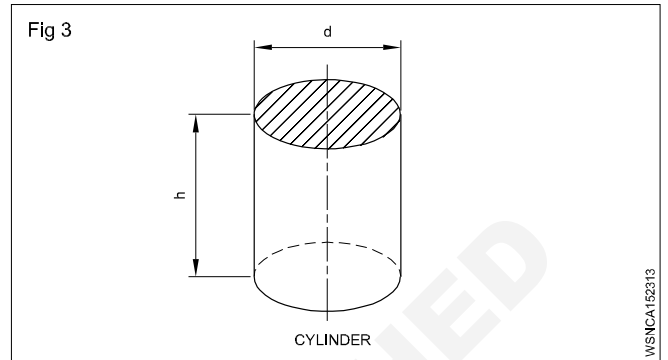


l = length, b = breadth and h = height

**Cylinder**

This is a prism whose top and bottom surfaces are equal and circular.

Volume of cylinder =  $\pi r^2 h$  or  $\frac{\pi}{4} d^2 h$



Curved area of cylinder =  $2\pi rh$  unit<sup>2</sup>

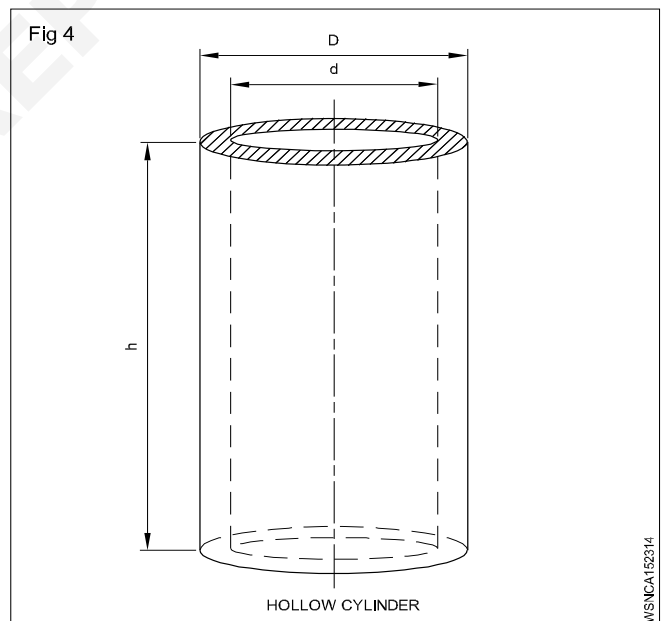
Total surface area of cylinder =  $2\pi r(h+r)$  unit<sup>2</sup>

r = Radius of base, d = Diameter of base

h = Height of cylinder

**Hollow cylinder**

Hollow means empty space. In hollow cylinder there is an empty place. Water pipe is an example of hollow cylinder.



Volume of hollow cylinder =  $\pi (R^2 - r^2) h$  (or)  
 =  $\pi (R + r) (R - r) h$  (or)  
 =  $\frac{\pi}{4} (D^2 - d^2) h$  unit<sup>3</sup>  
 =  $\frac{\pi}{4} (D + d)(D - d) h$

Total surface area of hollow cylinder =

Inner + outer curved area + area of top and bottom circular part

TSA :  $2\pi Rh + 2\pi rh + 2\pi(R^2 - r^2)$

R = outer radius

r = inner radius

D = outer diameter

d = inner diameter

h = height of cylinder

t = thickness

$$\text{Mean dia} = \frac{D-d}{2}$$

If thickness given then:

Volume of hollow cylinder =  $\pi \times$  mean dia  $\times$  thickness  $\times$  height

### Finding out volumes of solids

The space occupied by a body is known its volume. The volume of a body indicates the capacity to hold substance in it.

The general form of Lateral surface area Total surface area and Volume is :

**Lateral surface area** = perimeter of the base  $\times$  height

**Total surface area** = LSA + 2 (base area )

**Volume** = Area of base  $\times$  height

Important and commonly used solids are described below one after another:

### Cube

1 Find the diagonal, lateral surface area,, total surface area and volume of a cube of side 4.5 cm.

$$\text{side } a = 4.5 \text{ cm}$$

$$\begin{aligned} \text{diagonal } d &= \sqrt{3}a \text{ unit} \\ &= 1.732 \times 4.5 \\ &= 7.794 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{L.S.A} &= 4a^2 \text{ unit}^2 \\ &= 4 \times 4.5 \times 4.5 \\ &= 81 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{T.S.A} &= 6a^2 \text{ unit} \\ &= 6 \times 4.5 \times 4.5 \\ &= 121.5 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} V &= a^3 \text{ unit}^3 \\ &= 4.5 \times 4.5 \times 4.5 \\ &= \mathbf{91.125 \text{ cc.}} \end{aligned}$$

2 Calculate volume of a cube where side is 9 cm

$$a = 9 \text{ cm}$$

$$V = ?$$

$$\begin{aligned} V &= a^3 \\ &= 9 \times 9 \times 9 \\ &= \mathbf{729 \text{ cm}^3} \end{aligned}$$

3 Find out side of the cube if a cube has volume of 3375cm<sup>3</sup>.

$$V = 3375 \text{ cm}^3$$

$$a = ?$$

$$a^3 = 3375$$

$$a = \sqrt[3]{3375}$$

$$= \sqrt{3 \times 3 \times 3 \times 5 \times 5 \times 5}$$

$$= 3 \times 5$$

$$= \mathbf{15 \text{ cm}}$$

4 Find the side of a cube, if its surface area is 216 cm<sup>2</sup>

Surface area = T.S.A = 216 cm<sup>2</sup>

$$6a^2 = 216$$

$$a^2 = \frac{216}{6}$$

$$= 36$$

$$a = \sqrt{36}$$

$$= 6 \text{ cm}$$

5 Find the side of the square tank, if its height is 2 metre and has the capacity to hold 50,000 litre of water.

Height of square shape tank (h) = 2 m

$$\text{Capacity} = 50,000 \text{ litre}$$

$$1000 \text{ litre} = 1 \text{ m}^3$$

$$50,000 \text{ Litre} = \frac{50000}{1000}$$

$$= 50 \text{ m}^3$$

$$\text{Capacity of tank} = 50 \text{ m}^3$$

$$a^2 \times h = 50$$

$$a^2 \times 2 = 50$$

$$a^2 = \frac{50}{2} = 25 \text{ m}^2$$

$$a = \sqrt{25} = 5 \text{ m}$$

**Side of the square tank = 5 m**

### Rectangular Solid (or) Cuboid

1 Find its volume and T.S.A if a tank is 20 m long, 15 m broad and 12 m high.

$$l = 20 \text{ m}$$

$$b = 15 \text{ m}$$

$$h = 12 \text{ m}$$

$$v = ?$$

$$\text{T.S.A} = ?$$

$$\begin{aligned}\text{Volume } v &= lbh \text{ unit}^3 \\ &= 20 \times 15 \times 12 \\ &= 3600 \text{ m}^3\end{aligned}$$

$$\begin{aligned}\text{T.S.A} &= 2(lb + bh + hl) \text{ unit}^2 \\ &= 2((20 \times 15) + (15 \times 12) + (20 \times 12)) \\ &= 2(300 + 180 + 240) \\ &= \mathbf{1440 \text{ m}^2}\end{aligned}$$

- 2 Find out its height if the cross section is 260 mm length and 180 mm wide rectangular and the capacity of a fuel tank is 10500 cm<sup>3</sup>.

$$\begin{aligned}l &= 260\text{mm} = 26 \text{ cm} \\ b &= 180 \text{ mm} = 18 \text{ cm} \\ v &= 10500 \text{ cm}^3 \\ h &= ?\end{aligned}$$

$$\begin{aligned}l.b.h &= \text{volume} \\ 26 \times 18 \times h &= 10500\end{aligned}$$

$$\begin{aligned}h &= \frac{10500}{26 \times 18} \\ &= \mathbf{22.44 \text{ cm}}\end{aligned}$$

- 3 How many litres of water it can store if a water tank has the following dimensions length = 1 metre, width = 0.8 metre and height = 1.2 metre?

$$\begin{aligned}\text{Volume} &= l \times b \times h \text{ unit}^3 \\ &= 1 \times 0.8 \times 1.2 \\ &= 0.96 \text{ m}^3 \quad [1 \text{ m}^3 = 1000 \text{ litres}] \\ &= 0.96 \times 1000\end{aligned}$$

**= 960 litres of water can store in the tank.**

- 4 Find its volume if the base of a prism is a rectangle having 5m length, 4m breadth and the height of the prism is 15m.

The base of prism is rectangle

$$\begin{aligned}\text{Area of base} &= \text{length} \times \text{breadth} \\ &= 5 \times 4 \\ &= 20 \text{ square m}\end{aligned}$$

$$\begin{aligned}\text{Volume of prism} &= \text{Area of base} \times \text{Height} \\ &= 20 \times 15 \\ &= \mathbf{300 \text{ cm}^3}\end{aligned}$$

### Cylinder

- 1 Find the volume and total surface are of a cylinder having 9cm diameter and 15 cm height.

$$\begin{aligned}\text{diameter} &= 9 \text{ cm} \\ \text{radius } r &= 4.5 \text{ cm} \\ \text{height } h &= 15 \text{ cm} \\ \text{Volume } V &= ?\end{aligned}$$

$$\begin{aligned}\text{T.S.A} &= ? \\ V &= \pi r^2 h \text{ unit}^3 \\ &= \frac{22}{7} \times 4.5 \times 4.5 \times 15 \\ &= 954.4 \text{ cm}^3 \\ \text{T.S.A} &= 2\pi r(h+r) \text{ unit}^2 \\ &= 2 \times \frac{22}{7} \times 4.5 (15 + 4.5) \\ &= 2 \times \frac{22}{7} \times 4.5 \times 19.5 \\ &= \mathbf{551.4 \text{ cm}^2}\end{aligned}$$

- 2 Calculate the radius if the curved surface area of a cylindrical roller is 48π cm<sup>2</sup> and the roller is 10 cm long

$$\begin{aligned}\text{C.S.A} &= 48\pi \text{ cm}^2 \\ \text{length} &= 10 \text{ cm} \\ \text{radius} &= ? \\ 2\pi rh &= 48\pi \\ 2 \times \pi \times r \times 10 &= 48\pi \\ r &= \frac{48 \times \pi}{2 \times \pi \times 10} \\ &= \mathbf{2.4 \text{ cm}}\end{aligned}$$

- 3 Find its radius if the volume of a cylinder is 5544 cm<sup>3</sup> and its height is 16 cm.

$$\begin{aligned}\pi r^2 h &= v \\ 3.14 \times r^2 \times 16 &= 5544 \\ r^2 &= \frac{5544}{3.14 \times 16} \\ r^2 &= \frac{5544}{50.24} \\ &= 110.35 \\ r &= \sqrt{110.35} \\ &= \mathbf{10.5 \text{ cm}}\end{aligned}$$

- 4 Find the diameter of the tank if the volume of a circular tank is 68.46 m<sup>3</sup>, its height is 2 m.

$$\begin{aligned}\pi r^2 h &= 68.46 \\ r^2 &= \frac{68.46}{3.14 \times 2} \\ r^2 &= 10.9 \\ r &= \sqrt{10.9} \\ &= 3.3 \text{ m} \\ \text{diameter} &= 2r \\ &= 2 \times 3.3 \\ &= \mathbf{6.6 \text{ m}}\end{aligned}$$

- 5 A cylindrical vessel is to be made of 3 metre long and 1.9994 metre diameter. Calculate its surface area, if it is in a closed form on one end.

$$\begin{aligned} h &= 3\text{m} \\ d &= 1.9994\text{ m} \\ r &= 0.9997\text{ m} \end{aligned}$$

$$\begin{aligned} \text{T.S.A} &= \text{C.S.A} + \text{Base area} \\ &= 2\pi rh + \pi r^2 \\ &= \left(2 \times \frac{22}{7} \times 0.9997 \times 3\right) + \left(\frac{22}{7} \times 0.9997^2\right) \\ &= 18.85 + 3.14 \\ &= \mathbf{21.99\text{ m}^2} \end{aligned}$$

- 6 How many litres of water a cylinder of radius 75 cm and height 100 cm can hold.

$$\begin{aligned} V &= \pi r^2 h \text{ unit}^3 \\ &= 3.142 \times 75 \times 75 \times 100 \\ &= 1767375 \text{ cm}^3 \\ &= \frac{1767375}{1000} \quad [1000 \text{ cc} = 1 \text{ litre}] \\ &= \mathbf{1767.375 \text{ litres.}} \end{aligned}$$

- 7 Calculate the height of cylindrical tin if a closed rectangular box 40 cm long, 30 cm wide and 25 cm deep has the same volume as that of cylinder tin of radius 17.5 cm.

Volume of cylinder = Volume of rectangular box

$$\pi r^2 h = l \times b \times h$$

$$\frac{22}{7} \times 17.5 \times 17.5 \times h = 40 \times 30 \times 25$$

$$\begin{aligned} h &= \frac{40 \times 30 \times 25 \times 7}{22 \times 17.5 \times 17.5} \\ &= \frac{210000}{6737.5} \\ &= \mathbf{31.17 \text{ cm}} \end{aligned}$$

- 8 An oxygen cylinder is 15 cm in diameter and 100 cm in length. It is filled with gas under pressure so that every  $\text{cm}^3$  of the cylinder contains  $120 \text{ cm}^3$  of gas. How much cc of oxygen does this hold?

$$\begin{aligned} \text{Volume of cylinder} &= \pi r^2 h \text{ unit}^3 \\ &= \frac{22}{7} \times 7.5 \times 7.5 \times 100 \\ &= 17678.57 \text{ cm}^3 \end{aligned}$$

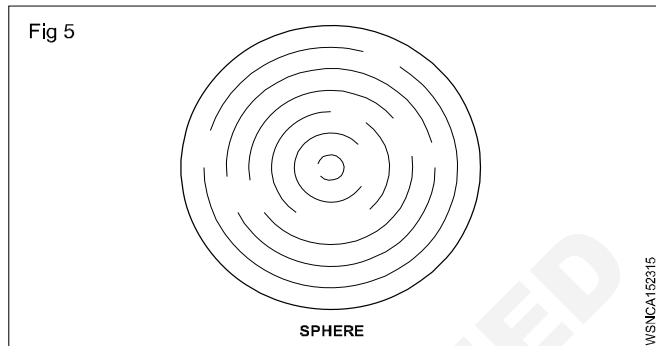
Gas contain in  $1 \text{ cm}^3 = 120 \text{ cm}^3$  of gas

$$\begin{aligned} \text{Gas contain in } 17678.57 \text{ cm}^3 &= 17678.57 \times 120 \\ &= 2121428 \text{ cm}^3 \end{aligned}$$

**Volume of oxygen = 2121428 cc.**

## Sphere

Sphere is a solid circular body.



$$\begin{aligned} \text{Volume of sphere} &= \frac{4}{3} \pi r^3 \quad \text{or} \\ &= \frac{\pi}{6} d^3 \text{ unit}^3 \end{aligned}$$

Total surface area of sphere =  $4\pi r^2 \text{ unit}^2$

Where r = Radius of sphere

d = Diametre of sphere

Radius =  $\frac{1}{2}$  of diameter

- 1 Find the volume and surface area of a sphere of 3 cm radius.

$$\begin{aligned} V &= \frac{4}{3} \pi r^3 \text{ unit}^3 \\ &= \frac{4 \times 22 \times 3 \times 3 \times 3}{3 \times 7} \end{aligned}$$

$$= 113.1 \text{ cm}^3$$

Total Surface Area =  $4\pi r^2 \text{ unit}^2$

$$\begin{aligned} &= 4 \times \frac{22}{7} \times 3 \times 3 \\ &= \mathbf{113.1 \text{ cm}^2} \end{aligned}$$

- 2 Find the diameter of sphere having volume of 15625 cc.

$$\frac{4}{3} \pi r^3 = \text{Volume}$$

$$\frac{4}{3} \times \frac{22}{7} \times r^3 = 15625$$

$$r^3 = \frac{15625 \times 3 \times 7}{4 \times 22}$$



$$= \frac{328125}{88}$$

$$= 3728.69$$

$$r = \sqrt[3]{3728.69}$$

$$= 15.51 \text{ cm}$$

$$\text{diameter} = 2 \times \text{radius}$$

$$= 2 \times 15.51$$

$$= 31.02 \text{ cm}$$

- 3 How many spherical balls of 1 cm radius can be made from a sphere of 32 cm diameter.

No. of balls x volume of small sphere = Volume of bigger sphere

$$N \times \frac{4}{3} \pi r^3 = \frac{4}{3} \pi R^3$$

$$N \times \frac{4}{3} \pi \times 1^3 = \frac{4}{3} \pi \times R^3$$

$$N = 16 \times 16 \times 16$$

$$= 4096 \text{ balls}$$

- 4 Three brass balls of diameters 3 cm, 4 cm and 5 cm are melted and make into one solid ball, if there is no wastage. Find the diameter of the solid ball.

$$1^{\text{st}} \text{ ball } d_1 = 3 \text{ cm,}$$

$$r_1 = 1.5 \text{ cm}$$

$$2^{\text{nd}} \text{ ball } d_2 = 4 \text{ cm,}$$

$$r_2 = 2 \text{ cm}$$

$$3^{\text{rd}} \text{ ball } d_3 = 5 \text{ cm,}$$

$$r_3 = 2.5 \text{ cm}$$

$$\text{Diameter of new ball} = ?$$

Volume of new ball = Volume of 3 spherical balls

$$\frac{4}{3} \pi r^3 = \frac{4}{3} \pi r_1^3 + \frac{4}{3} \pi r_2^3 + \frac{4}{3} \pi r_3^3$$

$$\frac{4}{3} \pi r^3 = \frac{4}{3} \pi (1.5^3 + 2^3 + 2.5^3)$$

$$r^3 = 3.375 + 8 + 15.625$$

$$r^3 = 27$$

$$r = \sqrt[3]{27}$$

$$r = \sqrt[3]{3 \times 3 \times 3}$$

$$r = 3 \text{ cm}$$

$$\text{Diameter of the ball} = 2 \times r$$

$$= 2 \times 3$$

$$= 6 \text{ cm}$$

## Assignment

### Cube

- Find the diagonal, lateral surface area, total surface area and volume of cube, whose side is 15 cm.
- Find the volume of 10 cubes where each side is 5 cm.
- Find its volume if a solid cube has each of its sides 60 mm long.
- What is its side if the total surface area of a cube is 384 m<sup>2</sup>.

### Cuboid

- Find the volume of the tank in m<sup>3</sup>, if the length is 60 m, breadth 40 m and height 20 m.
- Find the volume of a C.I. casting of a rectangular block having 25 cm x 20 cm x 8 cm size.
- Calculate the total surface area of a box whose length, width and height are 120 cm, 50 cm and 60 cm respectively.
- Find the volume of the sheet if a brass sheet is of 25 cm square and 0.4 cm thick.

### Cylinder

- Find the curved surface area of cylinder whose diameter is 18 cm and height 34 cm?
- Find the total surface area of cylinder whose diameter is 24 cm and height 40 cm?
- Find out the volume of cylinder whose base is 10 cm radius and height is 40 cm?

### Sphere

- Find the volume of sphere having diameter 3.5cm?
- Find the total surface area of a sphere having radius 1.75 cm?
- How many spherical balls of 1 cm radius can be made from a sphere of 16 cm diameter.
- Three balls of diameter 2m, 4cm and 6 cm are melted and made into one solid ball. If there is no wastage, find the diameter of solid ball.

Trigonometry - Measurement of angles

**Introduction:**

Trigonometry is the branch of mathematics which deals with the study of measurement and relationship of the three sides and three angles of a triangle.

**Units:**

**Measurement of Angles**

There are three systems of measuring the angle:

**(i) Sexagesimal System**

This is called British System. In this system, one right angle is divided into 90 equal parts which are called degrees. Each part is divided into 60 parts which are called minutes. Each minute is divided into 60 parts which are called seconds. The parts so divided respectively are called:

One degree (1°), one minute (1') and one second (1")

It means 1 right angle = 90° (90 degrees)

1 degree (1°) = 60' (60 minutes)

1 minute (1') = 60" (60 seconds)

In Trigonometry, mostly this system is used.

**(ii) Centesimal System**

This is called French System. In this system, the right angle is divided into 100 equal parts which are called grades. Each grade is divided into 100 minutes and each minute is divided into 100 seconds.

Parts so divided are respectively called:

One grade (1 g), one minute (1'), one second (1").

It means 1 right angle = 100 grades (100g)

1 grade (1 g) = 100 minutes (100')

1 minute (1') = 100 seconds (100")

90° = 100g (because each is a right angle)

This system is easier than Sexagesimal System. But to use this system many other systems will have to be devised that is why this system is not used.

**(iii) Circular System**

In this system, the unit of measuring angles is radian. It is that angle which is formed at the centre and is formed of an arc of length equal to radius in a circle.

There is one constant ratio between the circumference and dia of a circle. This is represented by π .

$$\frac{\text{Circumference}}{\text{Diameter}} = \text{constant point} = \pi$$

$$\begin{aligned} \text{Circumference} &= \pi \times \text{dia} \\ &= 2\pi r \text{ (where r is radius of the circle)} \end{aligned}$$

$$\pi = \frac{22}{7}$$

Circumference makes an angle (2πr) = 360°

Radius of the circle makes an angle (r) = 1 Radian

$$\text{ie : } \frac{C}{r} = \frac{360^\circ}{1\text{Radian}}$$

$$\frac{2\pi r}{r} = \frac{360^\circ}{1\text{Radian}}$$

$$2\pi = \frac{360^\circ}{1\text{Radian}}$$

$$2\pi \text{ Radian} = 360^\circ$$

$$\pi \text{ Radian} = 180^\circ$$

$$1 \text{ Radian} = \frac{180^\circ}{\pi}$$

$$1^\circ = \frac{\pi}{180^\circ} \text{ Radian}$$

**Examples**

1 Convert 45°36'20" into degree and decimal of degree.

$$60 \text{ second} = 1 \text{ minute}$$

$$20 \text{ second} = \frac{20}{60} = 0.333'$$

$$60 \text{ minute} = 1 \text{ degree}$$

$$36.333 \text{ minute} = \frac{36.333}{60} = 0.606^\circ$$

$$45^\circ 36' 20'' = 45.606^\circ$$

2 Convert 24.59° into degree, minute and second

$$1 \text{ degree} = 60 \text{ minute}$$

$$0.59 \text{ degree} = 0.59 \times 60 = 35.4'$$

$$1 \text{ minute} = 60 \text{ second}$$

$$0.4 \text{ minute} = 60 \text{ sec} \times 0.4$$

$$= 24''$$

$$\text{Therefore } 24.59^\circ = 24^\circ 35' 24''$$

3 Change 50°37'30" into degrees

By changing angle degrees into decimals

$$30'' = \frac{30}{60} = 0.50'$$

$$37'30'' = 37.5'$$

$$37.5' = \frac{37.5}{60} = 0.625^\circ$$

$$50^\circ 37' 30'' = 50.625^\circ$$

4 Convert  $23^{\circ}25'32''$  into radians

We know  $1^{\circ} = 60' = 3600''$

Therefore  $23^{\circ}25'32''$

$$= \left( 23 + \frac{25}{60} + \frac{32}{3600} \right) \text{ degrees}$$

$$= \frac{82800 + 1500 + 32}{3600}$$

$$= \frac{84332}{3600}$$

But  $180^{\circ} = \pi$  radians

Therefore  $23.4255$  degrees

$$= \frac{23.4255}{180} \pi \text{ radians}$$

$$= \frac{23.4255}{180} \times \frac{22}{7} \text{ radians}$$

$$= \mathbf{0.4089 \text{ radians}}$$

5 Convert  $87^{\circ}19'57''$  into Radian.

$$19'57'' = 19' + \frac{57''}{60}$$

$$= 19' + 0.95'$$

$$= 19.95'$$

$$87^{\circ}19.95' = 87^{\circ} + \frac{19.95'}{60}$$

$$= 87^{\circ} + 0.332^{\circ} = 87.33^{\circ}$$

$$1^{\circ} = \frac{\pi}{180} \text{ radian}$$

$$87.33^{\circ} = \frac{\pi}{180} \times 87.33 \text{ radian}$$

$$= 1.524 \text{ radian}$$

6 Convert  $67^{\circ}11'43''$  into Radian

$$11'43'' = 11' + \frac{43''}{60}$$

$$= 11' + 0.716'$$

$$= 11.72'$$

$$67^{\circ}11.72' = 67^{\circ} + \frac{11.72'}{60}$$

$$= 67^{\circ} + 0.195^{\circ}$$

$$= 67.2^{\circ}$$

$$1^{\circ} = \frac{\pi}{180} \text{ radian}$$

$$67.2^{\circ} = \frac{\pi}{180} \times 67.2 \text{ radian}$$

$$= 1.173 \text{ radian}$$

7 Convert  $\frac{4}{7} \pi$  radian into degrees

$$1 \text{ radian} = \frac{180}{\pi} \text{ degree}$$

$$\frac{4}{7} \pi \text{ radian} = \frac{180}{\pi} \times \frac{4}{7} \pi \text{ degree}$$

$$= 102.9 \text{ degree}$$

$$= 102^{\circ} 0.9 \times 60'$$

$$= 102^{\circ} 54'$$

8 Convert 0.8357 radian into degrees

$$1 \text{ radian} = \frac{180}{\pi} \text{ degree}$$

$$0.8357 \text{ radian} = \frac{180}{\pi} \times 0.8357 \text{ degree}$$

$$= 47.88^{\circ}$$

$$= 47^{\circ} 0.88 \times 60'$$

$$= 47^{\circ} 52.80'$$

$$= 47^{\circ} 52' 0.8 \times 60''$$

$$= 47^{\circ} 52' 48''$$

9 Convert 2.752 radian into degrees

$$1 \text{ Radian} = \frac{180}{\pi} \text{ degree}$$

$$2.7520 \text{ radian} = \frac{180}{\pi} \times 2.752 \text{ degree}$$

$$= 157.7^{\circ}$$

$$= 157.7^{\circ} \times 60'$$

$$= 157^{\circ} 42'$$

10 Convert  $\frac{3}{5} \pi$  radian into degrees

$$1 \text{ Radian} = \frac{180}{\pi} \text{ degree}$$

$$\frac{3}{5} \pi \text{ radian} = \frac{180}{\pi} \times \frac{3}{5} \pi \text{ degree}$$

$$= 108^{\circ}$$

## Assignment

### Convert into Degree

1 12 Radian

### Convert into Radians

2  $78^{\circ}$

3  $47^{\circ}20'$

4  $52^{\circ}36'45''$

5  $25^{\circ}38''$

### Convert into degree, minute and seconds

6  $46.723^{\circ}$

7  $68.625^{\circ}$

8 0.1269 Radians

9 2.625 Radians

10  $\frac{3}{5}$  Radians

Trigonometry - Trigonometrical ratios

Dependency

The sides of a triangle bear constant ratios for a given definite value of the angle. That is, increase or decrease in the length of the sides will not affect the ratio between them unless the angle is changed. These ratios are trigonometrical ratios. For the given values of the angle a value of the ratios

$\frac{BC}{AB}$ ,  $\frac{AC}{AB}$ ,  $\frac{BC}{AC}$ ,  $\frac{AB}{BC}$ ,  $\frac{AB}{AC}$  and  $\frac{AC}{BC}$  do not change even when the sides AB, BC, AC are increased to AB', BC' and AC' or decreased to AB'', BC'' and AC''.

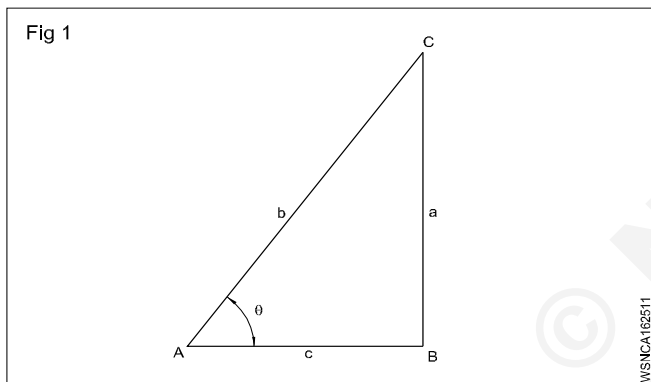
For the angle

AC is the hypotenuse

AB is the adjacent side

BC is the opposite side.

The ratios



The six ratios between the sides have precise definitions.

$$\text{Sine } \theta = \frac{BC}{AC} = \frac{\text{Opposite side}}{\text{Hypotenuse}} = \text{Sin } \theta$$

$$\text{Cosine } \theta = \frac{AB}{AC} = \frac{\text{Adjacent side}}{\text{Hypotenuse}} = \text{Cos } \theta$$

$$\text{Tangent } \theta = \frac{BC}{AB} = \frac{\text{Opposite side}}{\text{Adjacent side}} = \text{Tan } \theta$$

$$\text{Cosecant } \theta = \frac{AC}{BC} = \frac{\text{Hypotenuse}}{\text{Opposite side}} = \text{Cosec } \theta$$

$$\text{Secant } \theta = \frac{AC}{AB} = \frac{\text{Hypotenuse}}{\text{Adjacent side}} = \text{Sec } \theta$$

$$\text{Cotangent } \theta = \frac{AB}{BC} = \frac{\text{Adjacent side}}{\text{Opposite side}} = \text{Cot } \theta$$

Relationship between the ratios

$$\text{Cosec } \theta = \frac{AC}{BC} = \frac{1}{\frac{BC}{AC}} = \frac{1}{\text{sin } \theta}$$

$$\text{sec } \theta = \frac{AC}{AB} = \frac{1}{\frac{AB}{AC}} = \frac{1}{\text{cos } \theta}$$

$$\text{cot } \theta = \frac{AB}{BC} = \frac{1}{\frac{BC}{AB}} = \frac{1}{\text{tan } \theta}$$

$$\text{sin } \theta = \frac{\text{side BC}}{\text{side AC}} = \frac{a}{b}$$

$$\text{cos } \theta = \frac{\text{side AB}}{\text{side AC}} = \frac{c}{b}$$

$$\frac{\text{sin } \theta}{\text{cos } \theta} = \frac{\frac{a}{b}}{\frac{c}{b}} = \frac{a}{b} \times \frac{b}{c} = \frac{a}{c}$$

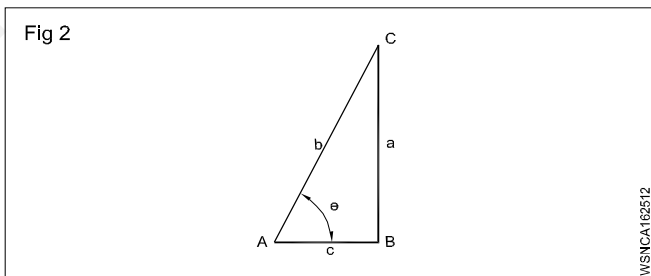
$$= \frac{\text{side BC}}{\text{side AB}} = \text{tan } \theta$$

$$\text{sin } \theta = \frac{1}{\text{cosec } \theta} \text{ or } \text{cosec } \theta = \frac{1}{\text{sin } \theta} \text{ or } \text{sin } \theta \cdot \text{cosec } \theta = 1$$

$$\text{cos } \theta = \frac{1}{\text{sec } \theta} \text{ or } \text{sec } \theta = \frac{1}{\text{cos } \theta} \text{ or } \text{cos } \theta \cdot \text{sec } \theta = 1$$

$$\text{tan } \theta = \frac{1}{\text{cot } \theta} \text{ or } \text{cot } \theta = \frac{1}{\text{tan } \theta} \text{ or } \text{cot } \theta \cdot \text{tan } \theta = 1$$

By pythagoras theorem we have,  $AC^2 = AB^2 + BC^2$



Dividing both sides of the equation by  $AC^2$ , we have

$$\frac{AC^2}{AC^2} = \frac{AB^2}{AC^2} + \frac{BC^2}{AC^2}$$

$$= \left[ \frac{AB}{AC} \right]^2 + \left[ \frac{BC}{AC} \right]^2$$

$$1 = (\text{cos } \theta)^2 + (\text{sin } \theta)^2$$

$$\text{sin}^2 \theta + \text{cos}^2 \theta = 1$$

Sine, Cosine, Tangent, Cosec, Sec and Cotangent are the six trigonometrical ratios

$$\text{tan } \theta = \frac{\text{Sin } \theta}{\text{Cos } \theta} \text{ and } \text{sin}^2 \theta + \text{cos}^2 \theta = 1$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

It can be transformed as

$$\sin^2 \theta = 1 - \cos^2 \theta$$

$$\sin \theta = \sqrt{1 - \cos^2 \theta}$$

$$\text{or } \cos^2 \theta = 1 - \sin^2 \theta$$

$$\cos \theta = \sqrt{1 - \sin^2 \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\sqrt{1 - \cos^2 \theta}}{\cos \theta}$$

$$\tan \theta = \frac{\sin \theta}{\sqrt{1 - \sin^2 \theta}}$$

We know  $\sin^2 \theta + \cos^2 \theta = 1$

Dividing both sides by  $\cos^2 \theta$ .

$$\frac{\sin^2 \theta}{\cos^2 \theta} + \frac{\cos^2 \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta}$$

$$\text{or } 1 + \tan^2 \theta = \sec^2 \theta$$

Using the same equation

$$\sin^2 \theta + \cos^2 \theta = 1.$$

Dividing both sides by  $\sin^2 \theta$ ,

$$1 + \frac{\cos^2 \theta}{\sin^2 \theta} = \frac{1}{\sin^2 \theta}$$

$$1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

### Trigonometrical Tables

| Ratio | 0° | 30°                  | 45°                  | 60°                  | 90° |
|-------|----|----------------------|----------------------|----------------------|-----|
| sin θ | 0  | $\frac{1}{2}$        | $\frac{1}{\sqrt{2}}$ | $\frac{\sqrt{3}}{2}$ | 1   |
| cos θ | 1  | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{2}$        | 0   |
| tan θ | 0  | $\frac{1}{\sqrt{3}}$ | 1                    | $\sqrt{3}$           | ∞   |

**When θ increases,**

**Sine value increases;**

**Cosine value decreases;**

**Tangent value increases to more than 1 when the angle is more than 45° ( $\tan 60^\circ = 1.732$ )**

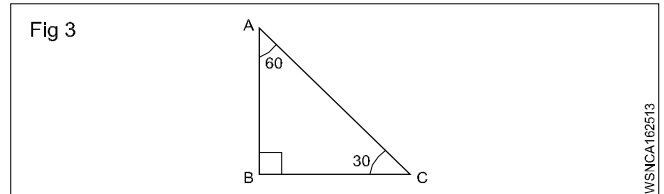
**Sine of an angle = Cosine of its complementary angle**

**Cosine of an angle = Sine of its complementary angle**

### Examples

If  $\sin 30^\circ = \frac{1}{2}$  find the value of  $\sin 60^\circ$

By applying pythagores theorem



$$BC^2 = AC^2 - AB^2$$

$$BC^2 = 2^2 - 1^2$$

$$= 4 - 1$$

$$= 3$$

$$BC = \sqrt{3}$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

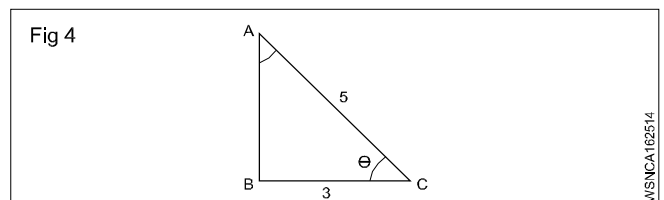
$\cos \theta = \frac{3}{5}$  Find the other trigonometrical ratios

By applying pythagores theorem

$$AB^2 = AC^2 - BC^2$$

$$= 5^2 - 3^2 = 25 - 9$$

$$= 16$$



$$AB = \sqrt{16} = 4$$

$$\text{Now } \sin \theta = \frac{4}{5}$$

$$\tan \theta = \frac{4}{3}$$

$$\operatorname{Cosec} \theta = \frac{5}{4}$$

$$\sec \theta = \frac{5}{3}$$

$$\cot \theta = \frac{3}{4}$$

## Signs of trigonometrical functions for angles more than 90°

| Ratio | 90 - θ | 90 + θ  | 180 - θ | 180 + θ | 270 - θ | 270 + θ | 360 - θ | - θ     |
|-------|--------|---------|---------|---------|---------|---------|---------|---------|
| sin   | cos    | cos     | sin     | - sin   | - cos   | - cos   | - sin   | - sin   |
| cos   | sin    | - sin   | - cos   | - cos   | - sin   | sin     | cos     | cos     |
| tan   | cot    | - cot   | - tan   | tan     | cot     | - cot   | - tan   | - tan   |
| cosec | sec    | sec     | cosec   | - cosec | - sec   | - sec   | - cosec | - cosec |
| sec   | cosec  | - cosec | - sec   | - sec   | - cosec | cosec   | sec     | sec     |
| cot   | tan    | - tan   | - cot   | cot     | tan     | - tan   | - cot   | - cot   |

**Simplify :**

$$\begin{aligned} \cot \theta + \tan (180+\theta) + \tan(90-\theta) + (\tan 360 - \theta) \\ = \cot \theta + \tan \theta - \cot \theta - \tan \theta \\ = 0 \end{aligned}$$

**Simplify :**

$$\begin{aligned} \frac{\cos (90+\theta) \sec (-\theta) \tan (180-\theta)}{\sec (360-\theta) \sin (180+\theta) \cos (90-\theta)} \\ = \frac{(-\sin \theta) \times (\sec \theta) \times (-\tan \theta)}{(\sec \theta) \times (-\sin \theta) \times (-\sin \theta)} \\ = \frac{\tan \theta}{\sin \theta} = \frac{1}{\cos \theta} = \sec \theta \end{aligned}$$

**simplify:**

$$\begin{aligned} \frac{\cos (90+\theta) \sec (-\theta) \tan (180-\theta)}{\sec (360-\theta) \sin (180+\theta) \cot (90-\theta)} \\ \cos (90+\theta) = -\sin \theta \\ \sec (-\theta) = \sec \theta \\ \tan (180-\theta) = -\tan \theta \end{aligned}$$

$$\sec (360-\theta) = \sec \theta$$

$$\sin (180+\theta) = -\sin \theta$$

$$\cot (90+\theta) = -\tan \theta$$

$$\begin{aligned} \frac{\cos (90+\theta) \sec (-\theta) \tan (180-\theta)}{\sec (360-\theta) \sin (180+\theta) \cot (90-\theta)} \\ = \frac{(-\sin \theta)(\sec \theta)(\tan \theta)}{(\sec \theta)(-\sin \theta)(-\tan \theta)} \\ = 1 \end{aligned}$$

**Simplify:**

$$\cot \theta + \tan (180+\theta) + \tan (90+\theta) + \tan (360-\theta)$$

$$\tan (180-\theta) = \tan \theta$$

$$\tan (90+\theta) = -\cot \theta$$

$$\tan (360-\theta) = -\tan \theta$$

$$\cot \theta + \tan (180+\theta) + \tan (90+\theta) + \tan (360-\theta)$$

$$\cot \theta + \tan \theta - \cot \theta - \tan \theta = 0$$

## Assignment

1 Given  $\sin 30^\circ = 1/2$ , find the value of  $\tan 60^\circ$

2 If  $\cos \theta = 4/5$ , find the other ratios

3 If  $\sin A = 3/5$ , find  $\cos \theta$ ,  $\tan \theta$  &  $\sec \theta$

4 If  $\tan \theta = 24/7$ , find  $\sin \theta$  and  $\cos \theta$

5 Find the value of  $\cos \theta$  and  $\tan \theta$ , if  $\sin \theta = 1/2$

6 If  $\cos \theta = 5/13$ , find the value of  $\tan \theta$

7 If  $\sin \theta = 1/2$ , find the value of  $\sin^2 \theta - \cos^2 \theta$

8 What is the value of

$$\frac{\sin^2 30^\circ}{\cos^2 45^\circ} + \frac{\tan 45^\circ}{\sec 60^\circ} - \frac{\sin 60^\circ}{\cot 45^\circ} - \frac{\cos 30^\circ}{\sin 90^\circ}$$

**Simplify :**

1  $\tan (90+A) + (\tan 180+A) \tan (90+A)$

2  $\frac{\cos(90+\theta) \cdot \sec(-\theta) \cdot \tan(180-\theta)}{\sec(360+\theta) \cdot \sin(180+\theta) \cdot \cot(90+\theta)}$